Intelligent Evolution Optimization: Guided from Deep Learning to Large Language Model

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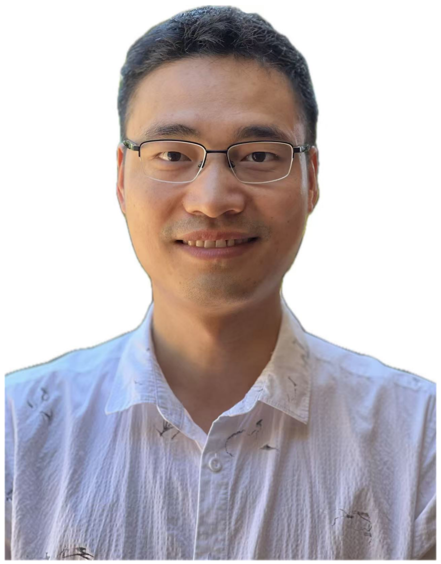
Hua Xu is a tenured Associate Professor in the Department of Computer Science at Tsinghua University. His research explores intelligent optimization and human-machine interaction in AI. He has published extensively in top venues, authored influential books such as Intelligent Evolutionary Optimization (Elsevier, 2024), and holds numerous patents. His work has received major honors including the National Science and Technology Progress Award and the Beijing Science and Technology Award, and he currently serves as Editor-in-Chief of Intelligent Systems with Applications.

Xiaodong Li is a Professor in the School of Computing Technologies at RMIT University, Melbourne. His research spans machine learning, evolutionary computation, swarm intelligence, and multiobjective optimization. He has served as Associate Editor for leading journals such as IEEE Transactions on Evolutionary Computation and Swarm Intelligence. A recipient of the ACM SIGEVO Impact Award and an IEEE Fellow, he has contributed extensively to the field through both research and leadership in IEEE CIS Task Forces.

Instructors

Yuan Sun is a Lecturer in Business Analytics and Artificial Intelligence at La Trobe University, Australia. He received his BSc in Applied Mathematics from Peking University, China, and his PhD in Computer Science from The University of Melbourne, Australia. His research interests include artificial intelligence, machine learning, operations research, and evolutionary computation. His research has contributed significantly to the emerging area of leveraging machine learning for combinatorial optimisation. He is the vice-chair of the IEEE task force on large-scale global optimisation and has organised special sessions and workshops, and delivered tutorials at the GECCO, PPSN, and CEC conferences.



Huigen Ye is a Ph.D. student at Tsinghua University, focusing on applying machine learning to accelerate large-scale optimization, particularly in mixed-integer programming. He has published papers in top conferences such as ICML, ICLR and AAAI. He is actively involved in academic service, serving as a reviewer for conferences like AISTATS, NeurIPS and ICLR.

Overview: Intelligent Evolution Optimization - Guided from Deep Learning to Large Language Models

Hua Xu

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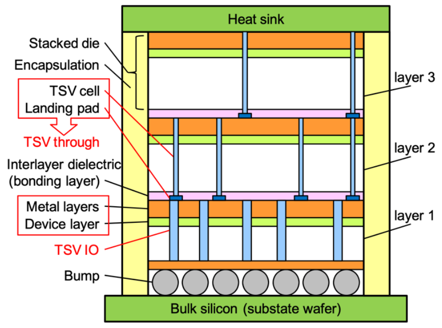
Contents

Background

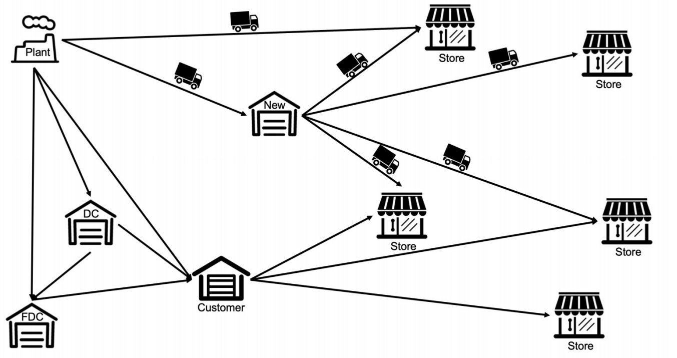
- **Several Optimization Cases**
  - 3D IC Partitioning Problem (Meitei et al., 2020)
  - Pickup and Delivery Problem with Time Windows (Dumas et al., 1991)
  - Supply Chain Management Problem (Villa, 2001)
- **Problem Definition**
  - Definition:

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \le 0, \quad i = 1, \ldots, m, \\ & h_j(x) = 0, \quad j = 1, \ldots, p \end{aligned}$$



(a) IC Partitioning Problem

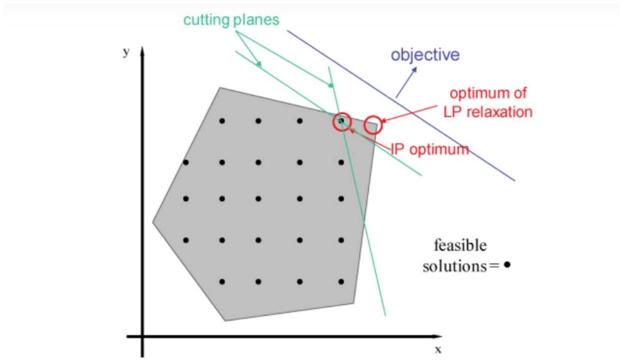
(b) Pickup and Delivery Problem



(c) Supply Chain Management

Background

- **Traditional Mathematical and Exact Methods**
    - **Convex Optimization Techniques:** Gradient Descent(AmariS, 1993), Newton's Method(Kelley, 2003)
    - **Solver:** SCIP (Achterberg, 2009), IPOPT (Biegler et al. 2009), Gurobi (Pedroso 2011), CPLEX (Bliek1ú et al., 2014)
    - **Academic Progress:** Advanced Branch-and-Bound Techniques (Morrison et al., 2016) , Cutting Plane Methods (Dey et al., 2018)
- **Challenges**
    - **Problem**: NP-hardness & High dimensionality
    - **Method**: Scalability Issues & Exponential Complexity



(a) Branch-and-Bound Techniques

GUROBI OPTIMIZATION

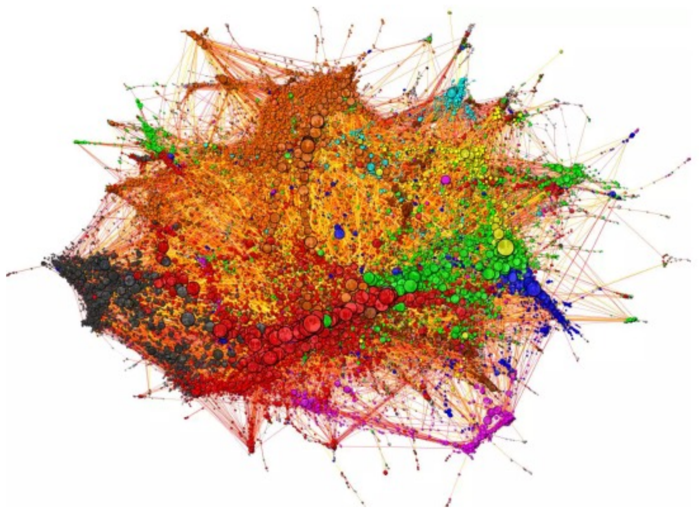
IBM CPLEX

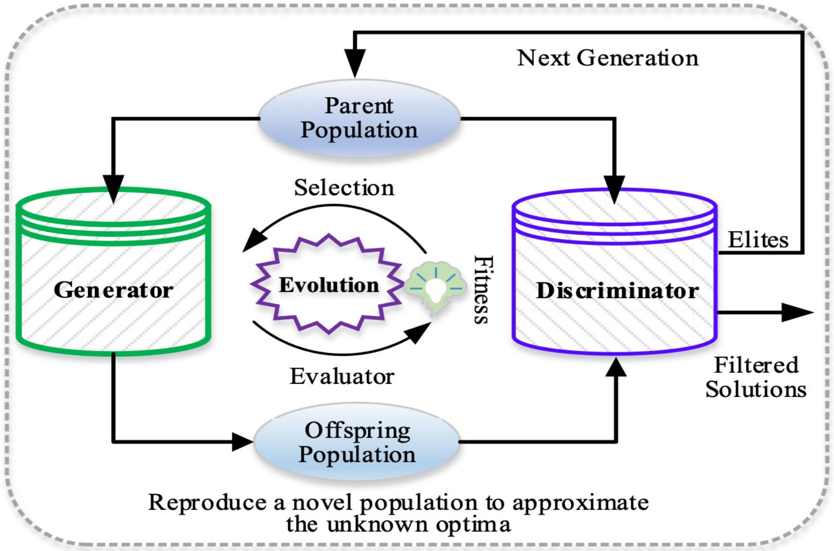
SCIP Solving Constraint Integer Programs

(b) Solvers

Background

- **Traditional Heuristics Method**
  - Large Neighborhood Search(LNS) (Song et al., 2020)
  - Adaptive Constraint Partition(Ye et al., 2023)
  - **Evolution Optimization Method**(Liu et al., 2023)
    - **Generator** — Evolution to Explore New Solutions
    - **Evaluator** — Assess Fitness of Solutions
    - **Discriminator** — Select the Best Solutions

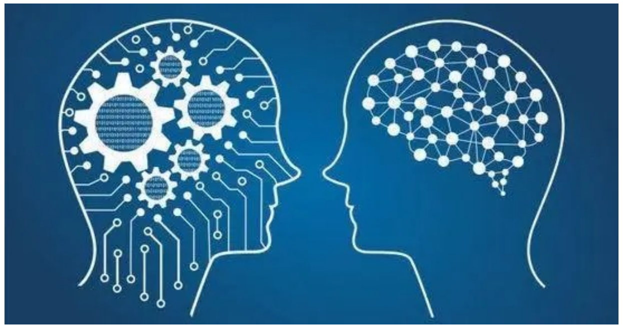

(a) Large Neighborhood Search



(b) Evolution Optimization Method

Background

- **Advantage**
  - Flexible and Effective Search Operators
  - Good Scalability for Problems
  - Parallelizable Population-based Search
- **Challenges**
  - Heavy Dependence on Careful Parameter Tuning
  - Reliance on Expert-designed Operators
  - Cold Start Issues and Slow Adaptation for New Problems

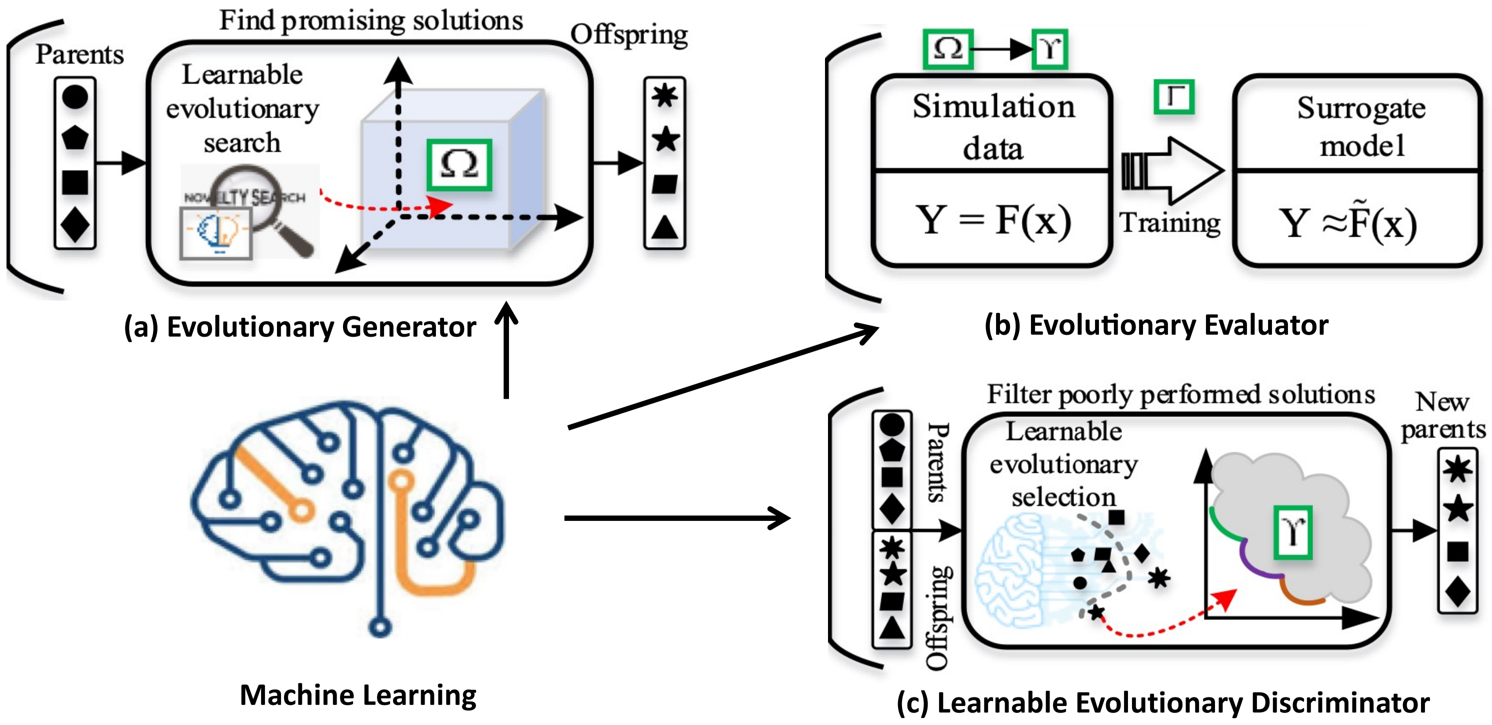


(a) Expert-designed Operators

(b) Cold Start Issues

Machine Learning Guided Evolutionary Optimization

- **Evolutionary processes assisted by machine learning**(Liu et al., 2023)
  - Evolutionary Generator
  - Evolutionary Evaluator
  - Learnable Evolutionary Discriminator
  - More details in **Topic I**

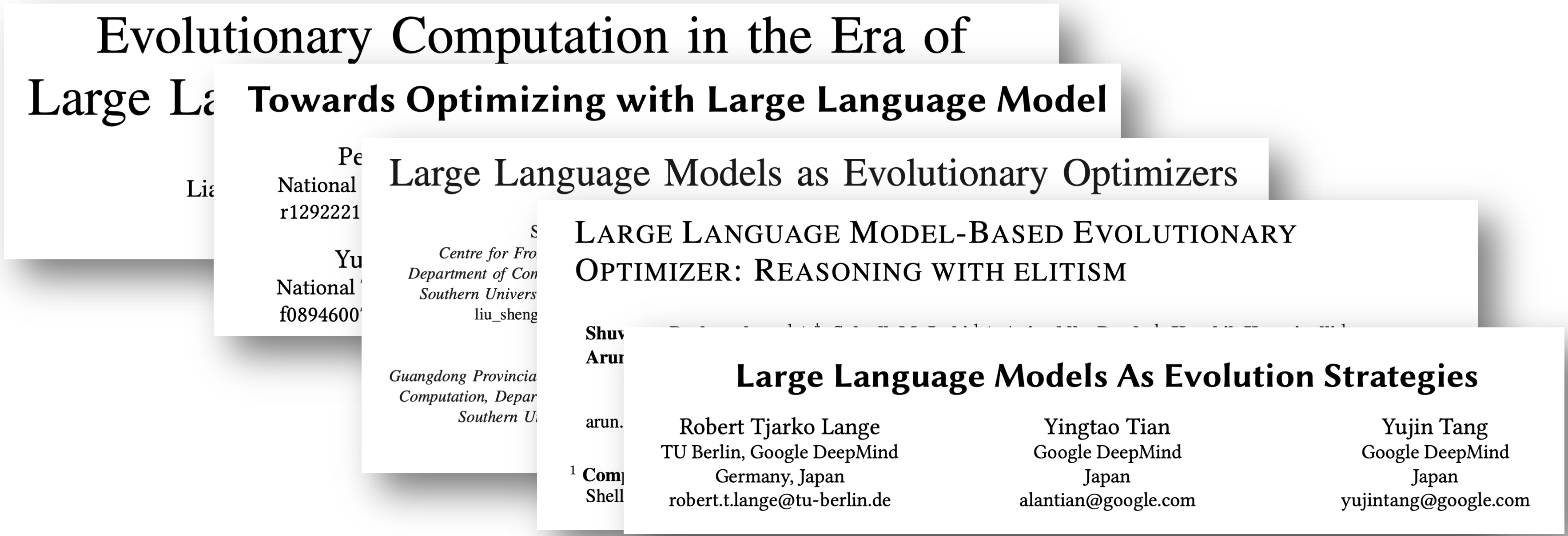

Find promising solutions
Parents
Learnable evolutionary search
Offspring
NOVELTY SEARCH
Ω
(a) Evolutionary Generator
Ω
ϒ
Simulation data
Y = F(x)
Γ
Training
Surrogate model
Y ≈F̃(x)
(b) Evolutionary Evaluator
Filter poorly performed solutions
New parents
Parents
Offspring
Learnable evolutionary selection
ϒ
(c) Learnable Evolutionary Discriminator
Machine Learning

Machine Learning Guided Evolutionary Optimization

- **Challenge**
  - Lack of diverse, large-scale training data for evolutionary learning
  - Current benchmarks are too small, simple, and fail to reflect real-world complexity
  - Traditional deep learning models are task-specific and require heavy retraining for each new optimization problem
- **What we need to do?**
  - Automated Data Generator(Yang et al., 2024)
  - Comprehensive Benchmark Test Suite(Ye et al., 2025a)
  - **Large Language Models (LLMs) to generalize and automate**

LLM Guided Evolutionary Optimization

- **Evolutionary processes assisted by LLM**



Evolutionary Computation in the Era of
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Towards Optimizing with Large Language Model

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Large Language Models as Evolutionary Optimizers

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LARGE LANGUAGE MODEL-BASED EVOLUTIONARY OPTIMIZER: REASONING WITH ELITISM

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Shell

Large Language Models As Evolution Strategies

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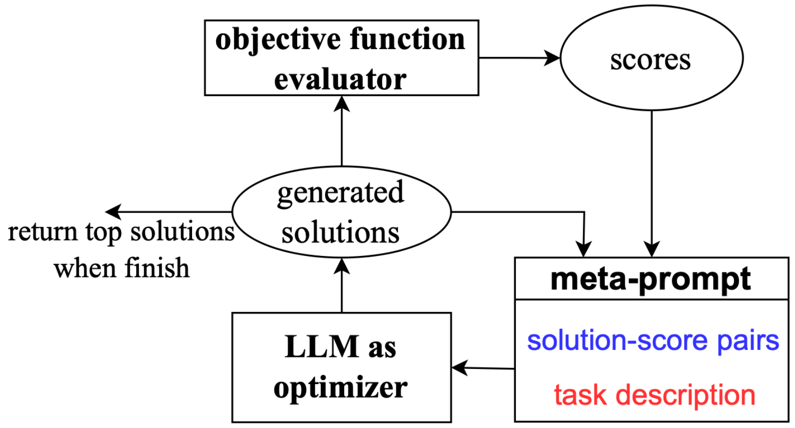
- **Key Direction**
  - LLM-assisted End-to-end Optimization
    - Automate both problem formulation and solving.
  - LLM-assisted Optimization Algorithm Generation
    - Generate heuristic operators for better search.

LLM Guided Evolutionary Optimization

- **LLM-assisted End-to-end Optimization**
  - LLMs show strong potential for black-box optimization.
    - **OPRO** (Yang et al., 2023): Iterative solution generation via optimization trajectories.
    - **LMEA** (Liu et al., 2024a): LLMs perform crossover and mutation in EA.
    - **LEO** (Brahmachary et al., 2025): Exploration and exploitation balanced through elitism.
  - More details in **Topic II**



(a) OPRO Framework

LLM Guided Evolutionary Optimization

- **LLM-assisted Optimization Algorithm Generation**
  - LLMs generate optimization algorithms beyond acting as operators.
    - Single-Round Generation:
      - **HybridMeta** (Pluhacek et al., 2023) : LLMs design hybrid metaheuristics by combining known methods.
      - **Optimus** (AhmadiTeshnizi et al., 2023) : LLMs automate MILP modeling, solving, and debugging.
    - Iterative Evolution:
      - **Funsearch**(Romera-Paredes et al., 2024): LLMs paired with evaluators to evolve interpretable programs solving combinatorial and algorithmic problems.
      - **EOH** (Liu et al., 2024b): LLMs co-evolve heuristic ideas and code structures.
      - **Reevo** (Ye et al., 2024): Reflective evolution enhances algorithm optimization via short- and long-term feedback.
  - More details in **Topic III**

The Tutorial Organization

- **Topic I**: Evolutionary Optimization Based on Machine Learning

  Prof. Xiaodong Li, IEEE Fellow, RMIT University

- **Topic II:** Business Optimization and Problem Formulation Using Large Language Models

  Prof. Yuan Sun, La Trobe University

- **Topic III:** Evolutionary Optimization Guided by Large Models

  Huigeng Ye, Tsinghua University

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Evolutionary Optimization Based on Machine Learning

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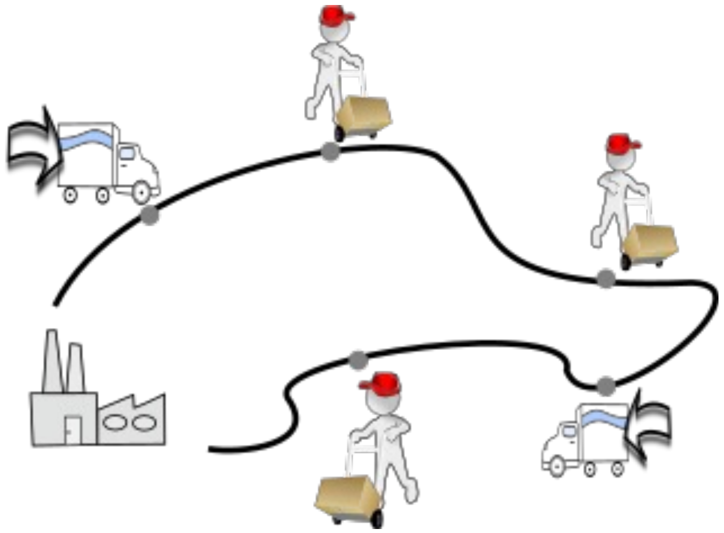
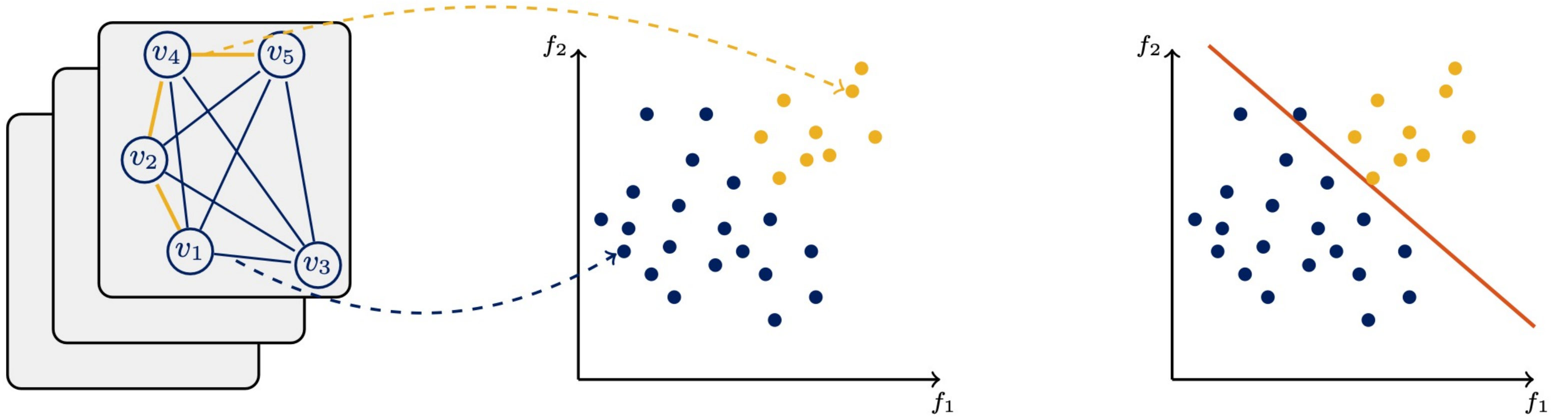
Why Can Machine Learning Help?

- Traditional optimization often relies on handcrafted rules for decision-making.
- Minor variations in problems frequently require redeveloping algorithms.
- In many industries, similar instances are solved repeatedly, often from scratch.
- There is an abundance of data, e.g., historical operations data collected over time.
- Collecting optimal solutions for training is feasible due to advanced solvers.
- Machine learning and deep learning are mature, powerful, and widely accessible.



Solution Prediction via Machine Learning: Training

- Solve a set of easy problem instances to optimality.
- Label decision variables using their optimal solution values.
- Extract features to characterize each decision variable.
- Train a machine learning model to predict optimal values for decision variables.



instances with optimal routes   mapping edges to feature space   learning a decision boundary

Features

A list of features that may be useful:

- Graph-based features: e.g., node weight, node degree, edge distance
- Mathematical features: e.g., objective bound (sum of weights of a node and its neighbors for maximum weighted clique problems)
- MIP formulation features: e.g., cost coefficients, number of non-zeros in the constraint matrix etc.
- LP relaxation features: e.g., reduced costs, optimal LP solution values
- Heuristic-based features: e.g., value-to-weight ratio in knapsack problems
- Statistical features from sample solutions: e.g., correlation between variable values and objective value, frequency of variable values in high-quality solutions (similar to Estimation of Distribution Algorithms or Ant Colony Optimization).

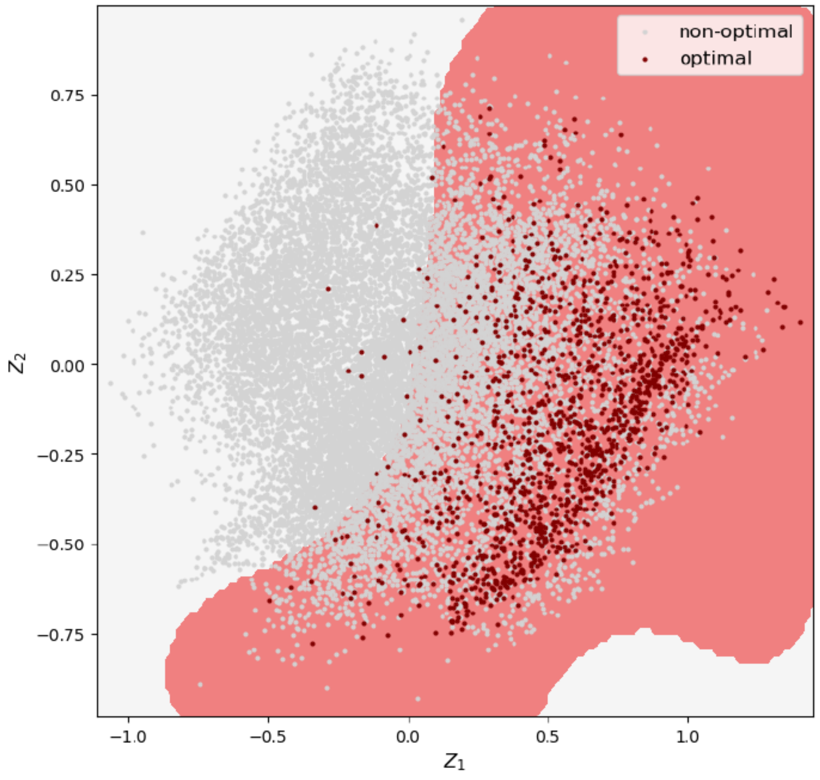
Machine Learning Models

Using Pre-defined Features

- Logistic Regression
- Support Vector Machines
- Deep Neural Networks

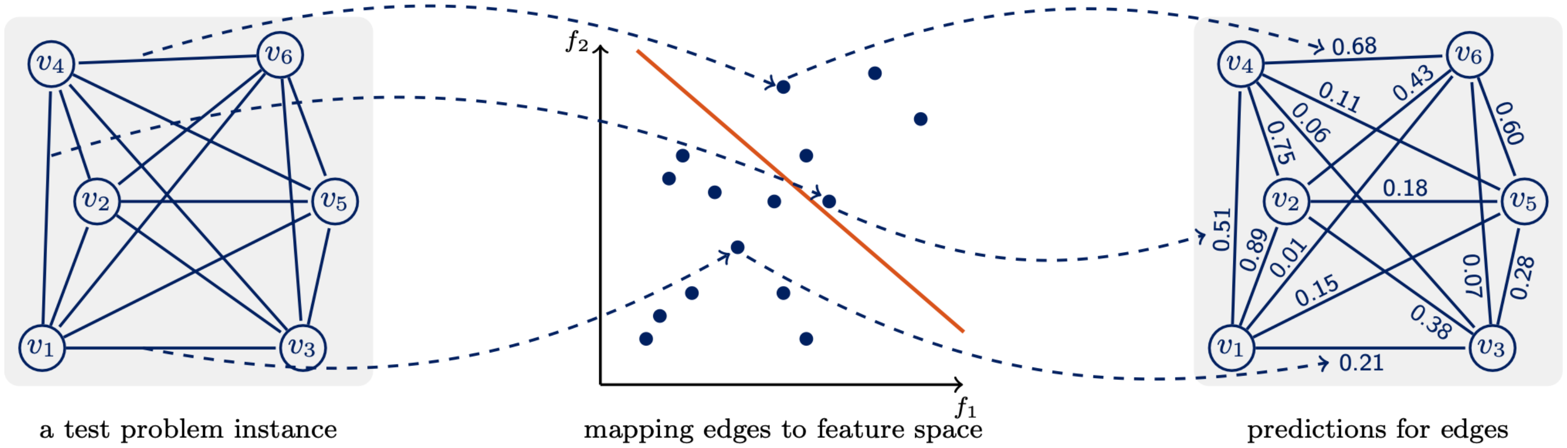
Automatically Learning Features

- Graph Neural Networks (GNNs): Leverage graph structure to learn node/edge-level representations
- Autoencoders: Learn compact, informative feature representations from raw inputs
- Transformers: Capture long-range dependencies, useful for sequences or sets



Solution Prediction via Machine Learning: Testing

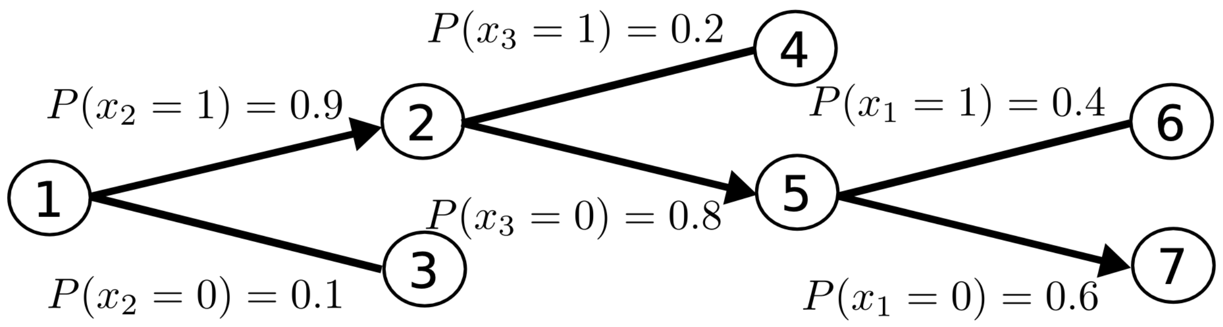
- Given a test problem instance, the trained machine learning model predicts the likelihood that each binary decision variable is part of the optimal solution (i.e., has a value of 1).



Note: Any bounded integer variable can be represented as a set of binary variables, making this approach broadly applicable.

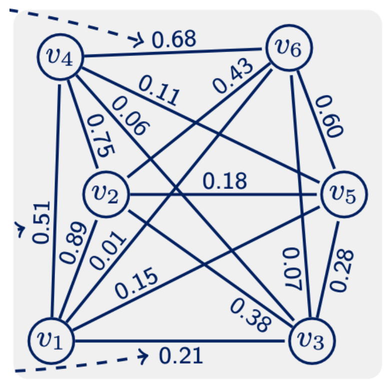
Constructing Solutions from ML Predictions

- Use predicted probabilities to greedily construct solutions, selecting at each step the variable most likely to be in the optimal solution.
- Expected to outperform hand-crafted heuristics, since such rules can be incorporated as features into the learning model.
- Can be combined with tree-based search methods (e.g., DFS) to prioritize high-quality regions of the solution space. (**NeurIPS'18, IJCNN'21**)(Li et al., 2018, Shen et al., 2021)

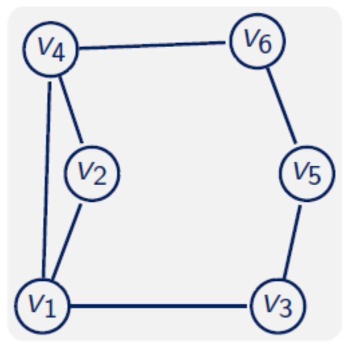


Pruning the Search Space with ML Predictions

- Fix or remove decision variables that are unlikely to be part of the optimal solution. Specifically, variables with predicted probabilities below a threshold can be fixed to 0. (**TPMAI'19, AAAI'19**)(Sun et al., 2019, Lauri et al., 2019)
- Apply a search algorithm to find a solution in the resulting reduced problem space, enabling faster and more focused optimization.



predictions for edges



removing low-quality edges

Extensions of ML-Guided Pruning

Constraint-Based Pruning (AAAI 2020)(Ding et al., 2020)

- Train a GNN to predict binary variable values in MIPs. Prune the search space via a global inequality constraint that limits deviation from the predicted solution.

Generalization to Unseen Instances (OR Spectrum 2021)(Sun et al., 2021)

- Train ML models on one category of instances and test across a variety of instances with different characteristics.

Multi-Stage Pruning (Journal of Heuristics, 2023) (Lauri et al., 2023)

- Apply the trained ML model recursively to prune the search space. Each stage trains a new classifier to progressively eliminate harder-to-prune elements.

ML-Guided Column Generation (ICLR 2023)(Sun et al., 2022a)

- Extend to problem formulations with exponentially many variables. Use the model to filter and select high-quality variables.

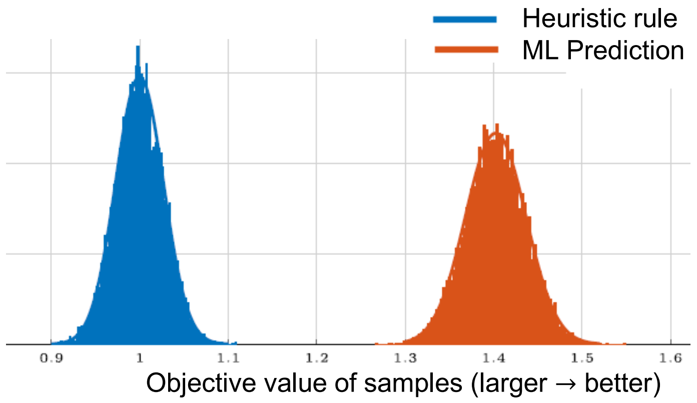
Reduce-Then-Optimize (Transportation Science, 2025)(Spieckermann et al., 2025)

- Use a GNN to identify a relevant subset of variables in the Fixed-Charge Transportation Problem (FCTP), reducing problem size and boosting solver efficiency.

Sampling Solutions Based on ML Predictions

At each step, the probability of selecting variable v_i is:

$$p_i = \frac{y_i}{\sum_{j \in S} y_j}$$



where:

- y_i is the ML prediction (likelihood of inclusion) for v_i
- S is the set of feasible candidate variables that can be added to the solution.

Sampling solutions for the pricing problem in Column Generation (AAAI'22)(Shen et al., 2022)

- A diverse set of high-quality solutions is required — not just one optimal solution.
- Compared to traditional sampling, this approach yields better-quality columns;
- Compared to exact or heuristic methods, it generates more high-quality solutions.

Boosting ACO with ML Predictions

Ant Colony Optimization (ACO) is a probabilistic algorithm that samples solutions using:

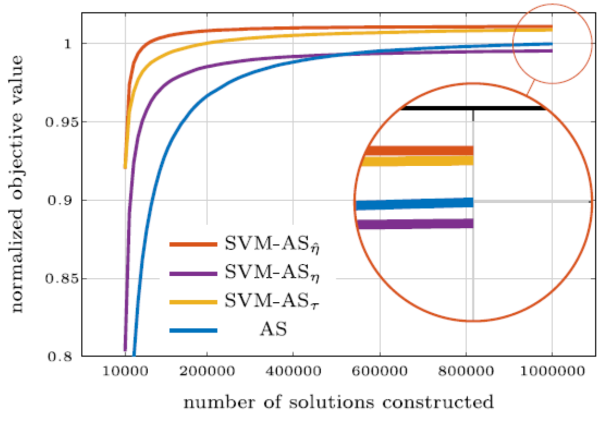
$$p_i = \frac{\tau_i \eta_i}{\sum_{j \in S} \tau_j \eta_j}$$

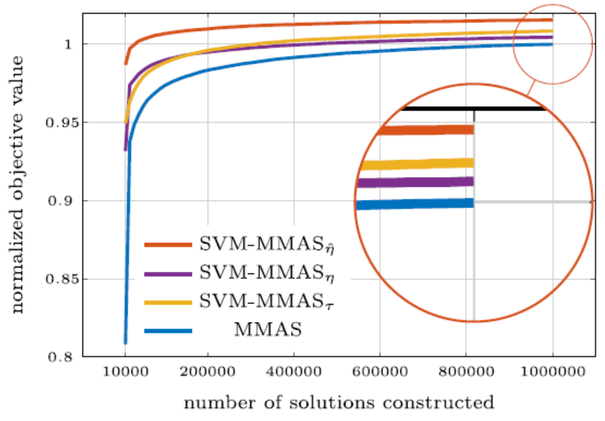
- $\boldsymbol{\eta}$: is heuristic rule,
- $\boldsymbol{\tau}$: pheromone trial reflecting the "evolved" quality of solution components.

Typically, $\boldsymbol{\tau}$ is initialized uniformly and $\boldsymbol{\eta}$ is set based on domain-specific heuristic rules.

ML-Enhanced Variants of ACO (COR 2022)(Sun et al., 2022a)

- SVM-ACO$_\eta$: Set $\eta_i = y_i$, where y_i is the ML predicted probability;
- SVM-ACOτ: Initialize $\tau_i = y_i$ using ML predictions;
- SVM-ACO$_{\hat{\eta}}$: Set the $\boldsymbol{\eta}$ value as a combination of ML predictions and a heuristic rule





Extensions of ML-Enhanced ACO

DeepACO (NeurIPS'23)(Ye et al., 2023)

- Use deep reinforcement learning to learn heuristic measures and is evaluated on eight combinatorial optimization problems.

LN-ACO (GECCO'23)(Liu et al., 2023)

- Use an "intelligent ant" with a pre-trained GNN to predict variable selection probabilities, forming a hybrid colony with traditional ants to guide the search process.

DLQ-ACO (GECCO'23)(Ramírez et al., 2023)

- Use GNN to generate variable selection probabilities and Q-learning to decide during solution construction whether to use ML-derived probabilities or traditional ACO heuristics.

ML-ACO for Column Generation (GECCO'24)(Xu et al., 2024)

- Incorporate ML predictions into the heuristic measures of ACO to efficiently generate multiple diverse, high-quality solutions for Column Generation.

GFACS (AISTATS'25)(Kim et al., 2025)

- Uses Generative Flow Networks to learn a multi-modal prior distribution to set the heuristic matrix for ACO, evaluated on seven combinatorial optimization problems.

Adaptive Solution Prediction via Machine Learning

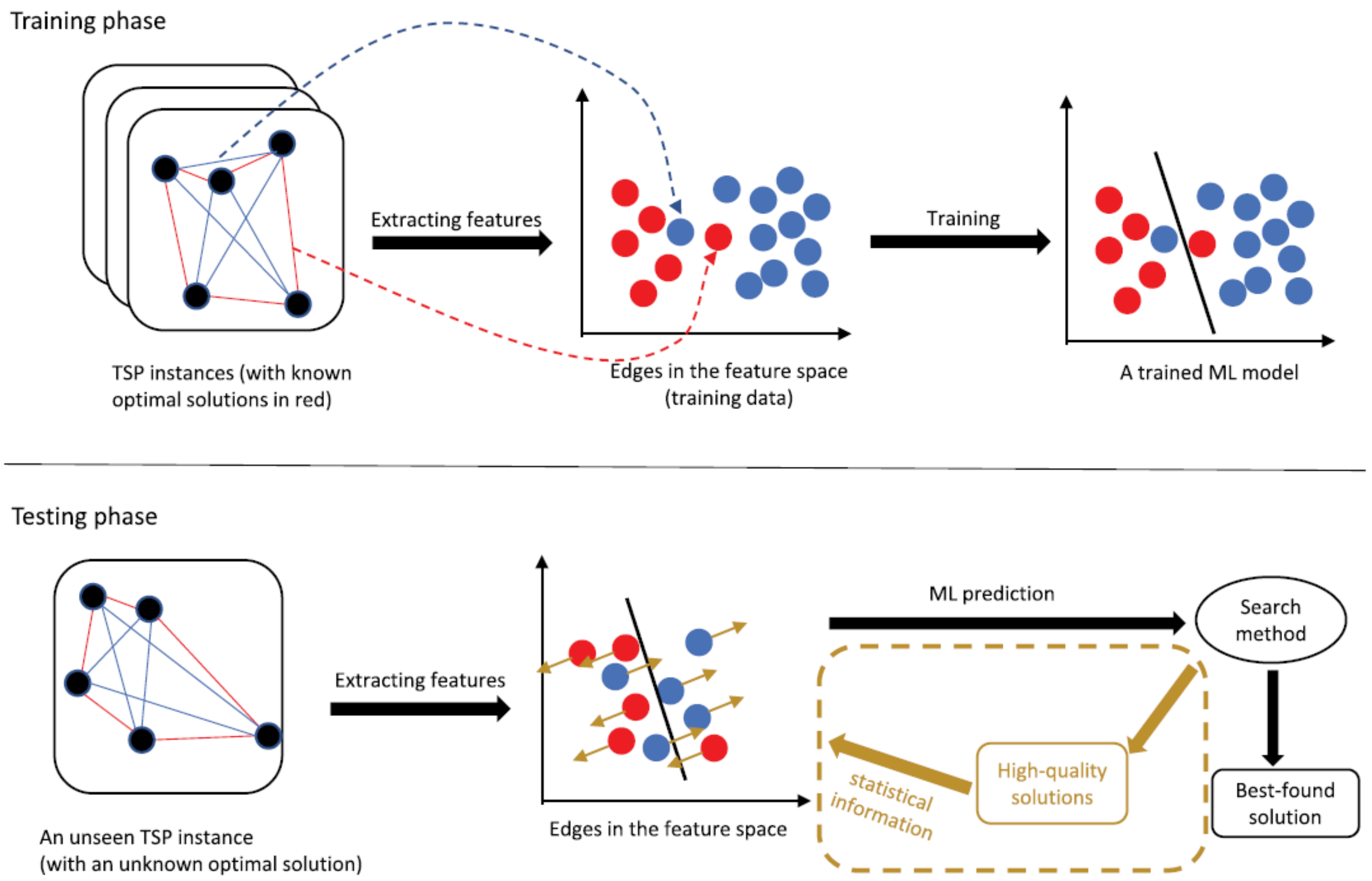
Limitations of Offline-Trained ML Models: Accuracy is often limited by

- Static, one-shot predictions that do not adapt during the search process.
- Challenges in crafting features that effectively represent decision variables.
- Distribution shift between training and testing instances.

Adaptive Solution Prediction (ASP) (EJOR 2023)(Shen et al., 2023)

- Refines ML predictions iteratively during the search on a problem instance.
- Incorporates feedback from search to progressively enhance prediction.
- Use statistical features extracted from sampled solutions to update predictions over time.
- Examples of statistical features include:
  - Correlation between variable values and objective value
  - Frequency of variables used in high-quality solutions (e.g., pheromone update in ACO)
- As the search generates better solutions, statistical features evolve, leading to more informed and accurate predictions.

Adaptive Solution Prediction via Machine Learning



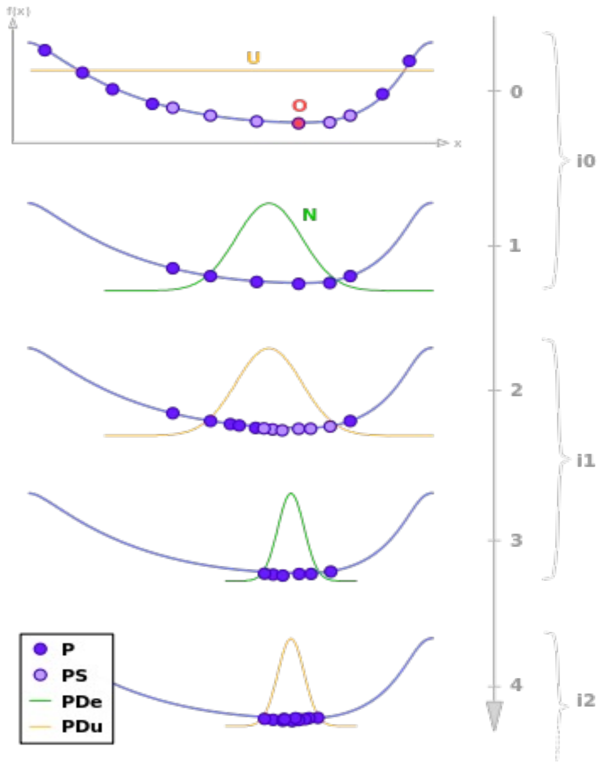
Relationship Between ASP and EDAs

Similarities

- Both sample the solution space of a specific instance.
- Both update a probability distribution to guide toward better solutions.
- Both exhibit online learning through iterative refinement.

Differences

- ASP uses an offline-trained ML model with knowledge from historical instances;
- EDAs (Estimation of Distribution Algorithms) typically learn from scratch during each run.
- ASP generalizes EDAs:
  - For example, ACO updates probabilistic mode (pheromone) using fixed rules.
  - ASP builds its prediction model from data using multiple features (can include ACO-style rules).



EDA builds and sample an explicit probabilistic model from a pool of promising candidate solutions (source: Wikipedia).

Relationship Between ASP and ML-ACO

Similarities

- Both integrate machine learning into metaheuristic search for combinatorial optimization.
- Both use ML predictions to guide solution construction or sampling.
- Both allow for adaptive behavior during problem-solving.

Differences

Aspect	ML-ACO	ASP
Integration Point	Injects ML into ACO components (pheromone, heuristic)	Predicts variable values to guide solution construction
Adaptivity	Often static once ML predictions are embedded	Dynamically refined using feedback from ongoing search
Learning Mode	Uses ML to improve ACO's components offline	Uses offline ML + online statistical adaptation
Generalization	Enhances specific ACO variants (e.g., SVM-ACO)	General framework applicable beyond ACO

Challenges and Future Directions

🚧 Key Challenges

- **Generalization Across Problems:** Building ML models that work across a class of problems or broader MILP remains difficult.
- **Feasibility & Guarantees:** ML predictions may violate constraints or yield infeasible solutions; providing optimality gap guarantee is challenging.
- **Refining Predictions:** Most ML models produce static, one-shot predictions. Online refinement and instance-specific adaptation are underexplored.

🚀 Future Directions

- **Expanding to Other Algorithms & Problems:** Apply ML to enhance e.g., PSO, GA, or handle dynamic and multi-objective optimization problems.
- **Cross-Domain Generalization:** Develop generic ML models using meta-learning, instance-space analysis, or domain-agnostic feature representations.
- **Adaptive Learning:** Move beyond ASP to integrate feedback (and possibly re-training) during search for better adaptability and ML predictions.
- **Exploring New ML Paradigms:** Leverage LLMs for heuristic code generation and MILP model formulation (see next sections).

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Topic II: Business Optimization and Problem Formulation Using Large Language Models

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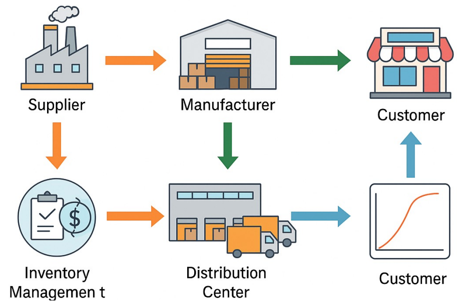
Melbourne, Australia

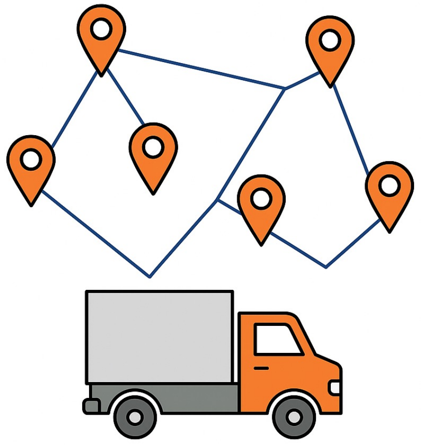
Business Optimization

Using mathematical models and analytical techniques to enhance decision-making, improve efficiency, reduce costs, and maximize profitability for a business.

Typical Business Optimization Problems:

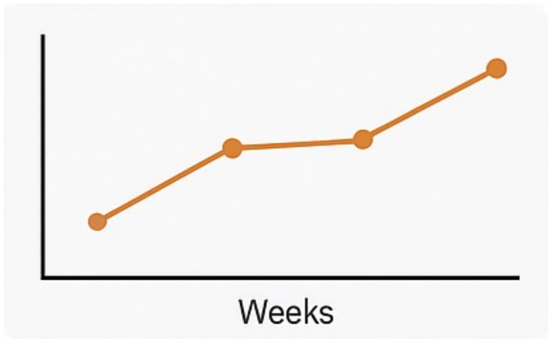
- Supply Chain Optimization
- Job Shop Scheduling
- Inventory Management
- Vehicle Routing Problem
- Production Planning
- Portfolio Optimization
- Staff Scheduling





Example

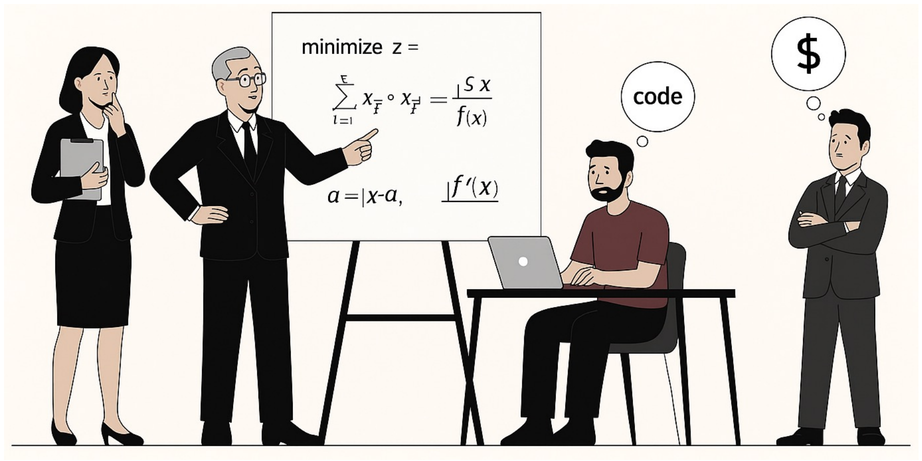
- I am a retailer selling laptops.
- Currently, there are 1015 laptops in the store.
- Each laptop in inventory incurs a cost of $1.0 per week.
- The forecast demand for laptops at the store in the next 8 weeks are {…}.
- Ordering laptops incurs a fixed cost of $2000 per order with up to 6000 laptops.
- What is the best plan to order laptops for the next 8 weeks to minimise the inventory and logistic costs while satisfying demand?



Challenges

Traditional way of applying optimization to solve business problems:

- Requires expert knowledge to translate natural language into solvable models
- Experts must have strong programming skills
- Interpretation of results and turning them into business actions is non-trivial



➢ Expertise is scarce, expensive, and often specialized to one technique.

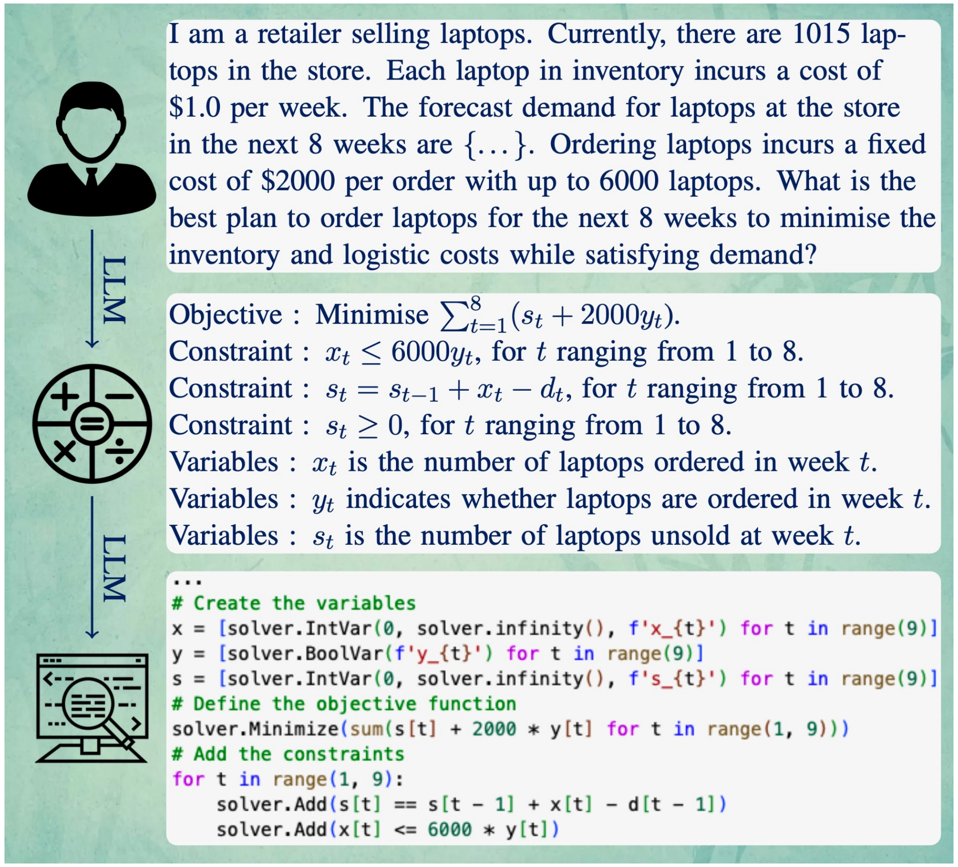
➢ Optimization process is time-consuming even for skilled professionals.

Automating Problem Formulation Using LLMs

Aim to use LLMs to automatically convert natural language descriptions of business problems into:

- Mathematical models
- Computer programs

This automation reduces reliance on expert knowledge and speeds up the optimization process [17]



I am a retailer selling laptops. Currently, there are 1015 laptops in the store. Each laptop in inventory incurs a cost of \$1.0 per week. The forecast demand for laptops at the store in the next 8 weeks are $\{\ldots\}$. Ordering laptops incurs a fixed cost of \$2000 per order with up to 6000 laptops. What is the best plan to order laptops for the next 8 weeks to minimise the inventory and logistic costs while satisfying demand?

Objective : Minimise $\sum_{t=1}^{8}(s_t + 2000y_t)$.
Constraint : $x_t \leq 6000y_t$, for t ranging from 1 to 8.
Constraint : $s_t = s_{t-1} + x_t - d_t$, for t ranging from 1 to 8.
Constraint : $s_t \geq 0$, for t ranging from 1 to 8.
Variables : x_t is the number of laptops ordered in week t.
Variables : y_t indicates whether laptops are ordered in week t.
Variables : s_t is the number of laptops unsold at week t.

```
...
# Create the variables
x = [solver.IntVar(0, solver.infinity(), f'x_{t}') for t in range(9)]
y = [solver.BoolVar(f'y_{t}') for t in range(9)]
s = [solver.IntVar(0, solver.infinity(), f's_{t}') for t in range(9)]
# Define the objective function
solver.Minimize(sum(s[t] + 2000 * y[t] for t in range(1, 9)))
# Add the constraints
for t in range(1, 9):
    solver.Add(s[t] == s[t - 1] + x[t] - d[t - 1])
    solver.Add(x[t] <= 6000 * y[t])
```

Early Efforts and Examples

🎓 Academic:

- Stanford University
- University of Cambridge
- Zhejiang University
- Chinese University of Hong Kong

Augmenting Operations Research with Auto-Formulation of Optimization Models from Problem Descriptions

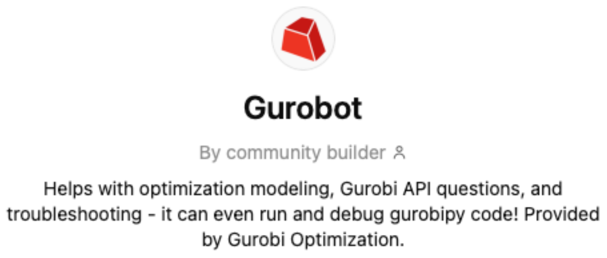
Rindranirina Ramamonjison, Haley Li, Timothy T. Yu, Shiqi He, et al.
EMNLP 2022

OptiMUS: Scalable Optimization Modeling with (MI)LP Solvers and Large Language Models

Ali Ahmaditeshnizi, Wenzhi Gao, Madeleine Udell
ICML 2024

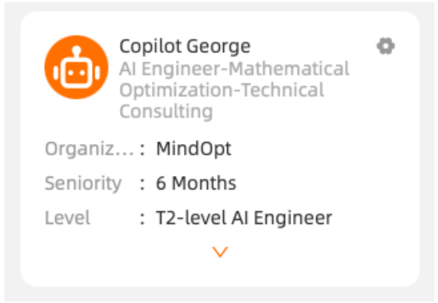
🏢 Industry:

- Huawei
- Gurobi
- Alibaba
- Microsoft


Gurobot

By community builder

Helps with optimization modeling, Gurobi API questions, and troubleshooting - it can even run and debug gurobipy code! Provided by Gurobi Optimization.


Copilot George
AI Engineer-Mathematical Optimization-Technical Consulting

Organiz...	:	MindOpt
Seniority	:	6 Months
Level	:	T2-level AI Engineer

Method Overview

🧠 **Prompt-Based Techniques**

- Use carefully crafted prompts to guide LLMs in translating natural language into optimization models.

🔄 **Prompt Optimization**

- Iteratively refine prompts based on model output and feedback to improve formulation accuracy.

🔧 **Fine-Tuning Approaches**

- Train LLMs on domain-specific datasets (fully or efficiently) to enhance performance and consistency.

🤖 **Workflow Architectures**

- Structure the problem-solving process using single-agent or multi-agent systems for modular reasoning.

🔍 **Retrieval-Augmented Generation (RAG)**

- Dynamically fetch and insert relevant past examples or context to improve task relevance and accuracy.

Zero-Shot Prompting

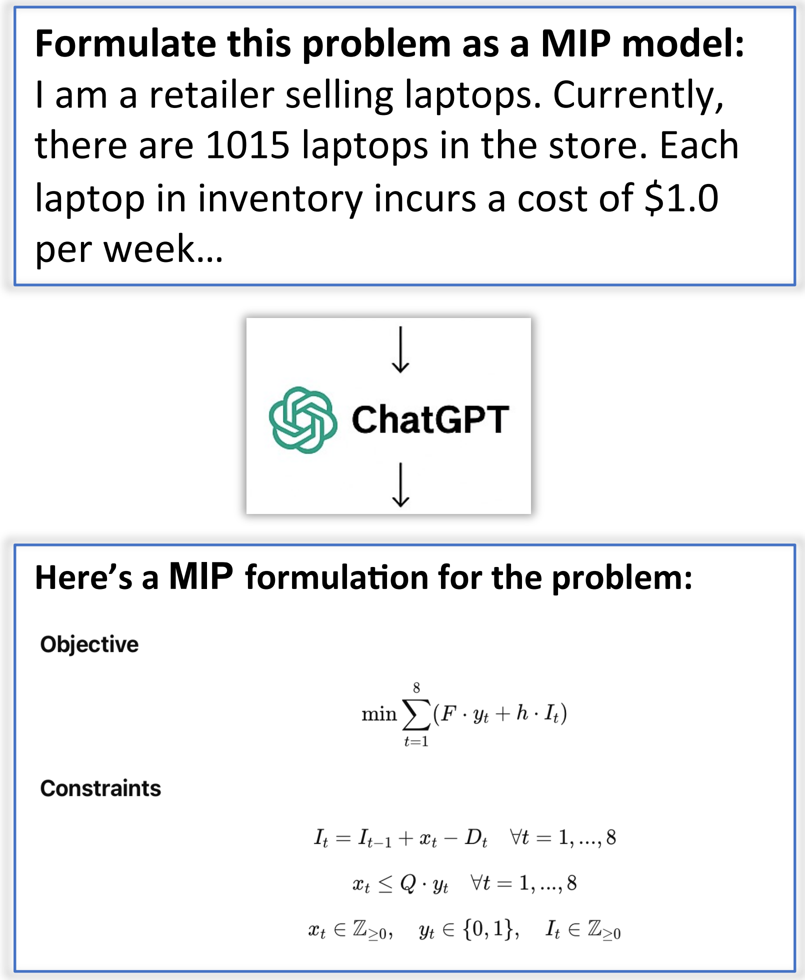
Use a LLM to formulate an optimization model from natural language with **no** examples [1]

✅ **Strengths:**

- Easy to use and implement
- No labelled data or fine-tuning
- Compatible with commercial LLMs

⚠️ **Limitations:**

- Accuracy may drop on complex tasks
- Highly sensitive to prompt phrasing



Formulate this problem as a MIP model:
I am a retailer selling laptops. Currently, there are 1015 laptops in the store. Each laptop in inventory incurs a cost of $1.0 per week…

Here's a MIP formulation for the problem:

Objective

$$\min \sum_{t=1}^{8} (F \cdot y_t + h \cdot I_t)$$

Constraints

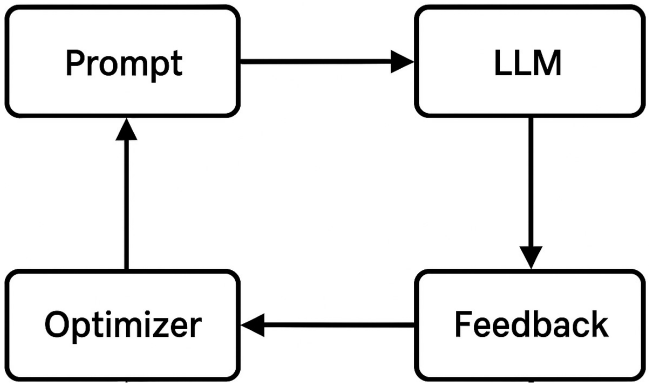
$$I_t = I_{t-1} + x_t - D_t \quad \forall t = 1, ..., 8$$
$$x_t \leq Q \cdot y_t \quad \forall t = 1, ..., 8$$
$$x_t \in \mathbb{Z}_{\geq 0}, \quad y_t \in \{0, 1\}, \quad I_t \in \mathbb{Z}_{\geq 0}$$

Prompt Optimization

- ➢ The process of automatically improving prompts to maximize LLM performance on a task.
- ➢ Altering the wording or structure of prompts to better align with model expectations.

🧪 Notable Methods:

- OPRO[2] – Large Language Models as Optimizers: Uses LLMs to propose and refine prompts iteratively based on performance feedback.
- EvoPrompt[3] – LLMs + Evolutionary Algorithms: Applies evolutionary search to discover high-performing prompts through generations.



🎯 Benefit:

Automates the search for high-quality prompts.

Few-Shot Prompting

- LLM is shown a **few** examples of input-output pairs before being asked to solve a new task.
- Demonstrates how a problem description maps to formulation.

✅ **Benefits:**

- Enables LLMs to learn task patterns
- Potentially more accurate than zero-shot prompting

⚠️ **Limitations:**

- Limited by prompt token length (can only include a few examples)
- Performance highly depends on quality and diversity of examples

Few-Shot Prompt

Example

Q: A bakery makes two items: bread and cake. Bread takes 1 hour to bake and cake takes 2 hours. The oven is available for 40 hours per week. Bread earns $4 profit, and cake earns $7. Formulate an integer linear programming model to maximize profit.

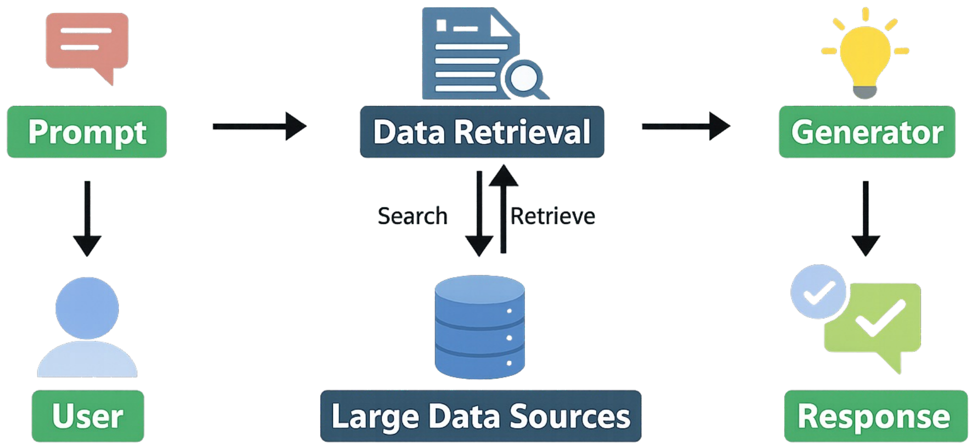
A:
$$\begin{aligned} \max \quad & 4B + 7C \\ \text{s.t.} \quad & B + 2C \leq 40 \\ & B, C \in \mathbb{N}_0 \end{aligned}$$

New Task

Q: A company produces products A and B. Each A requires 2 hours of labor, each B requires 1 hour. With 100 total labor hours available and profits of $30 for A and $20 for B, formulate a MIP model to maximize profit.

Retrieval-Augmented Generation (RAG)

- ➢ Enhances LLMs by retrieving relevant documents and inserting them into prompts.
- ➢ Helps handle complex or domain-specific tasks with limited LLM internal knowledge.



🚛 **DRoC (Decomposed Retrieval of Constraints)[4] :**

- Decomposes complex optimization problems into individual constraints.
- Retrieves relevant documentation/code for each constraint.
- Merges retrieved content to assist LLMs in generating accurate solver-compatible programs.

Chain-of-Thought Prompting

Encourages LLMs to reason step-by-step before generating the final formulation[5]

Mimics human thought processes by decomposing complex tasks into logical steps

Chain-of-Thought Prompt

Example

Q: A bakery makes two items: bread and cake. Bread takes 1 hour to bake and cake takes 2 hours. The oven is available for 40 hours per week. Bread earns \$4 profit, and cake earns \$7. Formulate an integer linear programming model to maximize profit.

A: 1) Let B = number of bread loaves, C = number of cakes.
2) These must be non-negative integers.
3) Objective: Maximize profit → 4B + 7C
4) Constraint: Bread takes 1 hour, cake takes 2 → $B + 2C \leq 40$

Final model:

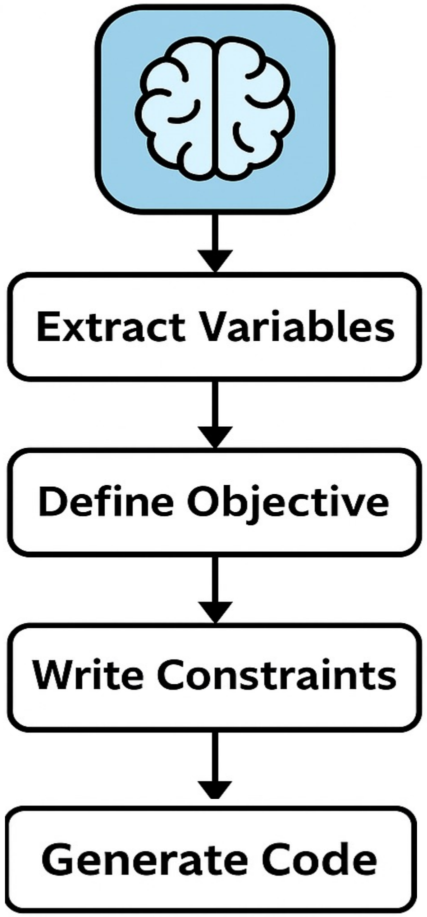
$$\begin{aligned} \max \quad & 4B + 7C \\ \text{s.t.} \quad & B + 2C \leq 40 \\ & B, C \in \mathbb{N}_0 \end{aligned}$$

New Task

Q: A company produces products A and B. Each A requires 2 hours of labor, each B requires 1 hour. With 100 total labor hours available and profits of \$30 for A and \$20 for B, formulate a MIP model to maximize profit.

Single-Agent Workflow

- A single LLM agent is used to handle the entire problem formulation task in a modular, step-by-step manner.
- The LLM is prompted to sequentially complete subtasks: extract variables, define objective, write constraints, and generate code.
- More scalable for long problems due to reduced context length
- Compatible with prompt-based or fine-tuned models



Multi-Agent Workflow

- ➢ Inspired by expert collaboration and chain-of-responsibility paradigms.
- ➢ Multiple LLMs or role-specialized agents collaborate in a structured workflow to perform different subtasks in problem formulation.
- ➢ Roles may include: Conductor (Coordinator), Interpreter, Formulator, Programmer, and Validator.

✅ **Advantages**:

- Improved performance via task specialization
- Easier to debug or refine each step/agent

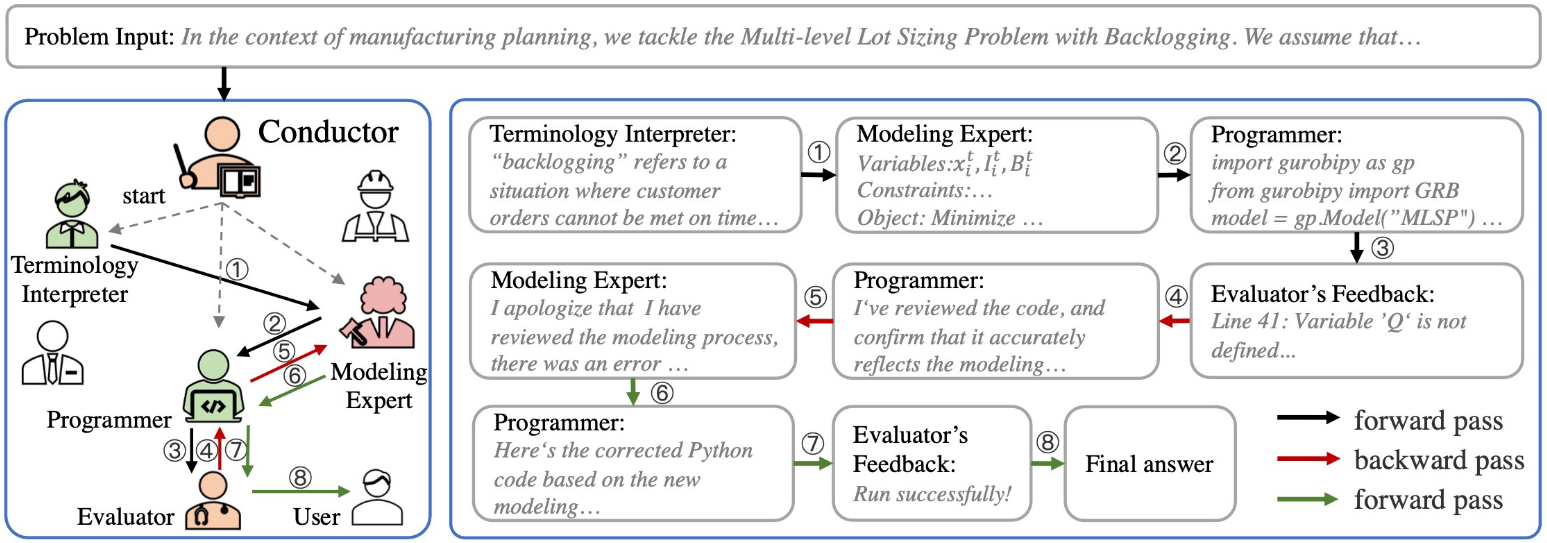
⚠️ **Limitations**:

- Higher complexity and resource-intensive (compute, cost, latency)
- Requires careful design of communication between agents

Multi-Agent Workflow Example

Chain-of-Experts (CoE)[6]

- A central Conductor coordinates specialized LLM agents (e.g., Interpreter, Modeler, Programmer).
- Agents collaborate iteratively to analyze, formulate, and verify optimization models.
- Combines forward reasoning (expert-driven modeling) with backward reflection (feedback-based revision).

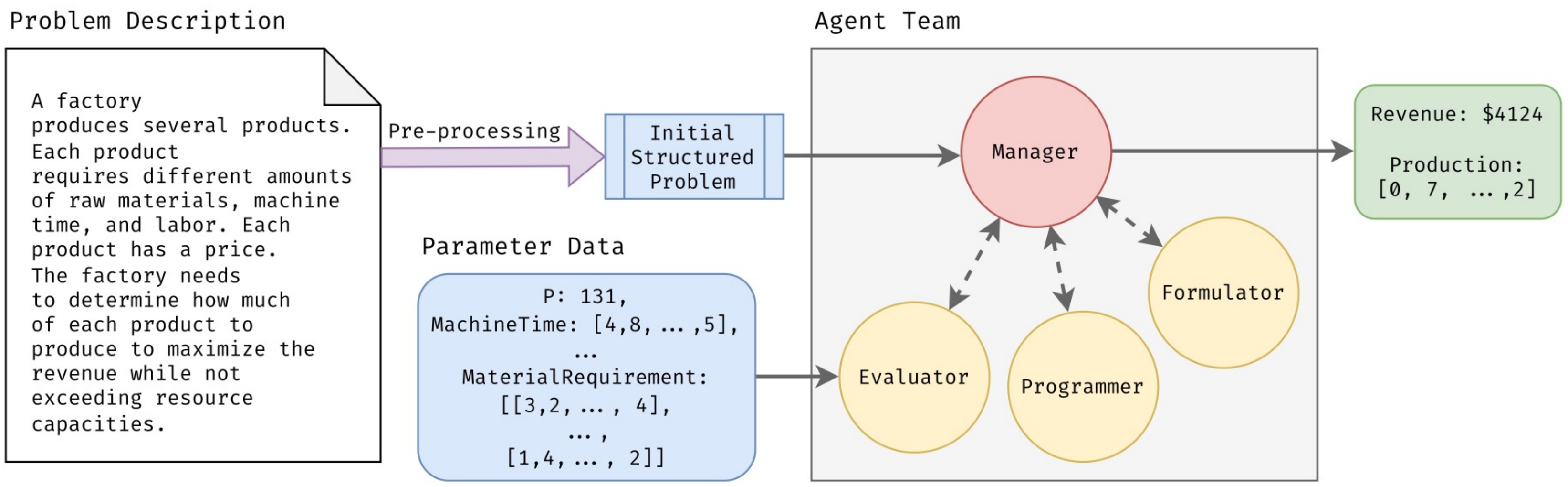


Problem Input: In the context of manufacturing planning, we tackle the Multi-level Lot Sizing Problem with Backlogging. We assume that…
Conductor
start
Terminology Interpreter
Modeling Expert
Programmer
Evaluator
User
①
②
③
④
⑤
⑥
⑦
⑧
Terminology Interpreter: "backlogging" refers to a situation where customer orders cannot be met on time…
Modeling Expert: Variables: x_i^t, I_i^t, B_i^t Constraints:… Object: Minimize …
Programmer: import gurobipy as gp from gurobipy import GRB model = gp.Model("MLSP") …
Evaluator's Feedback: Line 41: Variable 'Q' is not defined...
Programmer: I've reviewed the code, and confirm that it accurately reflects the modeling…
Modeling Expert: I apologize that I have reviewed the modeling process, there was an error …
Programmer: Here's the corrected Python code based on the new modeling…
Evaluator's Feedback: Run successfully!
Final answer
forward pass
backward pass
forward pass

Multi-Agent Workflow Example

OptiMUS [7]

- Manager: Oversees the workflow and coordinates agent interaction.
- Preprocessor: Extracts variables, objectives, and constraints from text.
- Formulator: Converts each clause into formal math (e.g., LaTeX).
- Programmer: Generates and debugs solver code.
- Evaluator: Runs code and checks for correctness.

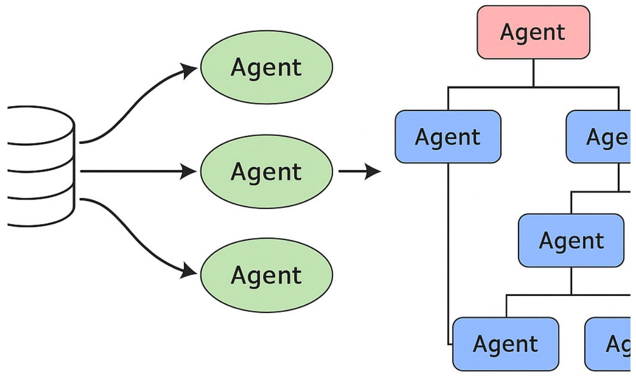

Problem Description
A factory produces several products. Each product requires different amounts of raw materials, machine time, and labor. Each product has a price. The factory needs to determine how much of each product to produce to maximize the revenue while not exceeding resource capacities.
Pre-processing
Initial Structured Problem
Parameter Data
P: 131, MachineTime: [4,8, … ,5], … MaterialRequirement: [[3,2, … , 4], … , [1,4, … , 2]]
Agent Team
Manager
Formulator
Evaluator
Programmer
Revenue: $4124
Production: [0, 7, … ,2]

Agentic Workflow Optimisation

- ➢ Traditional multi-agent LLM workflows require manual design of agent roles and communication strategies.
- ➢ Agentic workflow optimisation aims to automatically generate and refine multi-agent workflows for complex tasks like problem formulation.
- ➢ Enhances scalability, adaptability, and performance without hand-engineered pipelines.

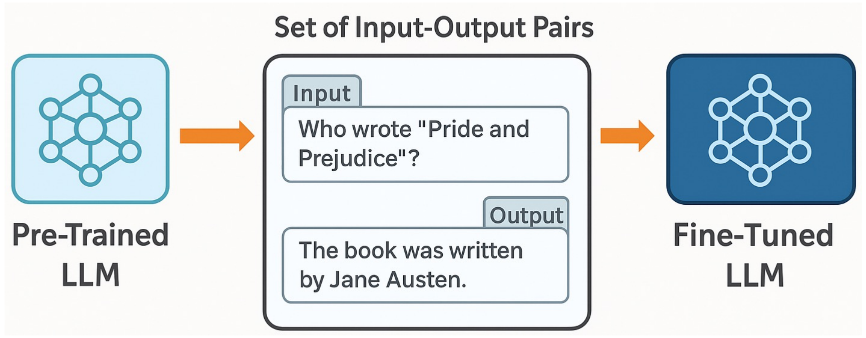
🚀**AFLOW[8]: Automating Agentic Workflow Generation**

- Represents workflows as graphs of modular, reusable LLM-invoking nodes.
- Uses Monte Carlo Tree Search (MCTS) to explore and optimize workflows.



Fine-Tuning

- ➢ Refers to training a pre-trained language model on a specific dataset of input-output pairs.
- ➢ Input: problem description
- ➢ Output: formulation & code


Set of Input-Output Pairs
Pre-Trained LLM
Input
Who wrote "Pride and Prejudice"?
Output
The book was written by Jane Austen.
Fine-Tuned LLM

✅ **Strengths**

- Improves performance on domain-specific or complex tasks
- More reliable and consistent than prompt-only methods

⚠️ **Limitations**

- Requires labelled datasets (e.g., NL + LP/MIP pairs)
- Computationally expensive and time-consuming

Fine-Tuning Examples

Full Fine-Tuning: Updates all model parameters on domain-specific data

Efficient Fine-Tuning: Updates a subset of parameters or adds lightweight modules:

- LoRA [13] – Low-rank adaptation to attention weights
- Adapters – Trainable modules inserted into the transformer stack

Recent Efforts

- LM4OPT [1] : Fine-tunes LLaMA-2-7B to convert natural language into optimization models using the NL4Opt dataset.
- OptLLM [9] : Trains Qwen-based agents with multi-turn interaction and solver feedback for optimization modeling.
- ORLM [10] : Fine-tunes several open-source 7B-scale LLMs using the open-instruct framework and OR-Instruct data.
- LMBO [11] : Applies full fine-tuning to real-world production scheduling, demonstrating improved task performance and code generation.
- LLMOPT [12] : Combines structured five-part representations with supervised fine-tuning for general-purpose optimization modeling.

Benchmark Datasets

Evaluate the capability and accuracy of LLMs in formulating optimization models

Support fine-tuning and instruction alignment for task specialization

🧪 **Existing Benchmark Datasets:**

- NL4Opt [14] : 1101 annotated LP problems across 6 domains and tasks.
- Mamo [15] : 863 MILP problem instances (652 Easy, 211 Complex)
- NLP4LP [7] : 65 textbook-sourced LP and MILP instances
- ComplexOR [6] : 37 expert-annotated OR problems from diverse real-world sources
- IndustryOR [10] : 100 real-world OR problems, 5 types, 3 difficulty levels
- OPTIBENCH [16] : 605 verified problems covering LP, NLP, MIP, and tabular data
- Sched [11] : Two scheduling datasets with 1,700 instances via modular expansion.

Synthetic Datasets

Motivation

- Lack of large-scale, high-quality labelled datasets for optimization tasks.
- Manual annotation is expensive, time-consuming, and requires domain expertise.

🔧 Synthetic Dataset & Approach

- **LLMOPT** [12] augments 1,763 seed problems using diverse instruction templates via GPT-4, followed by expert filtering and detailed labeling.
- **OR-Instruct** [10] **:** Generated by expanding 686 seed cases using GPT-4, structured prompts, augmentation (e.g., rephrasing, constraint variation), and human filtering to ensure correctness.
- **RESOCRATIC-29K** [16] : Generated by reverse-constructing optimization problems from formulations, then translating and filtering them for correctness and diversity.

Challenges and Future Directions

⚠️ **Challenges**

- Ambiguity and missing details in natural language descriptions
- Vast and diverse space of business optimization problem types
- Limited availability of high-quality, labeled datasets for training and evaluation
- Prompt sensitivity, hallucination, and generation of solver-incompatible models
- High cost and complexity of running or accessing large language models

🚀 **Future Directions**

- Scalable synthetic data generation (e.g., ReSocratic) for robust training
- Enriched benchmarks covering various real-world business optimization tasks
- Enhanced prompt optimization and retrieval-augmented generation methods
- Agentic workflow automation (e.g., AFLOW) for multi-agent coordination
- Integration of validation tools and solver feedback for self-debugging
- Development of cost-efficient LLMs with strong optimization performance

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Evolutionary Optimization Guided by Large Models

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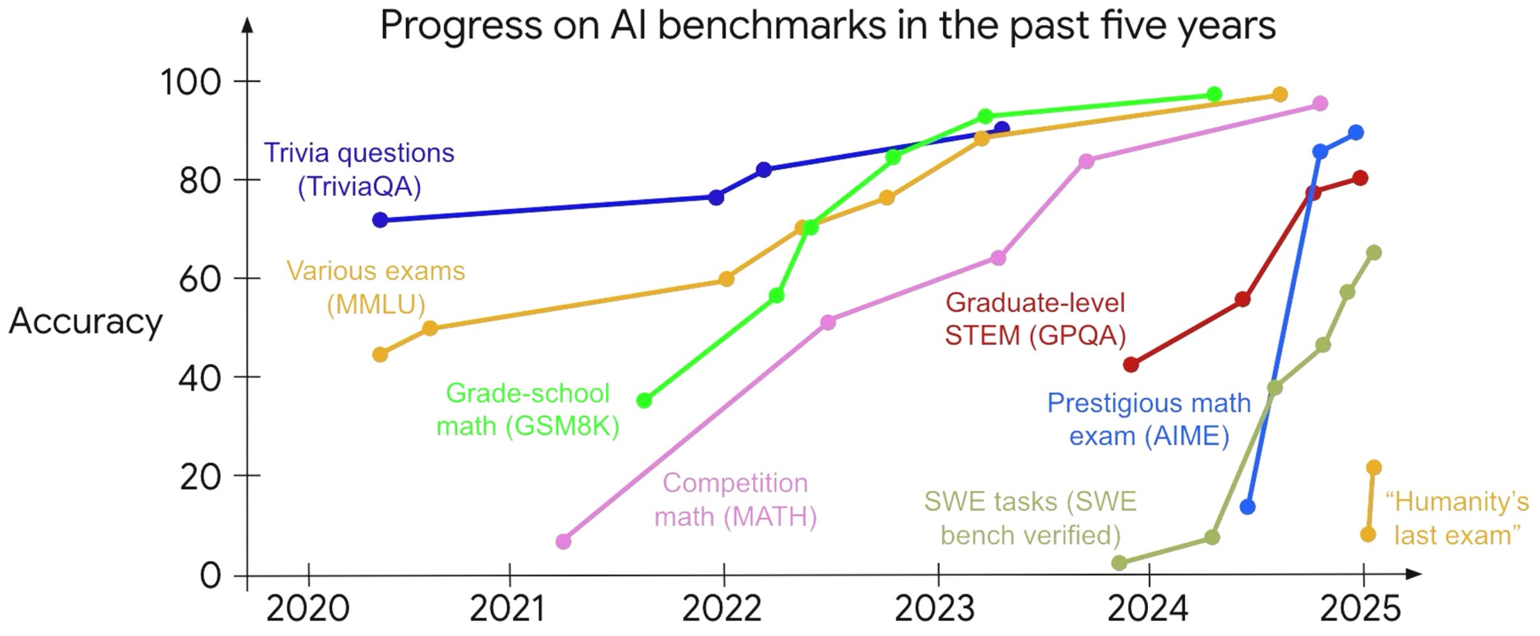
Background

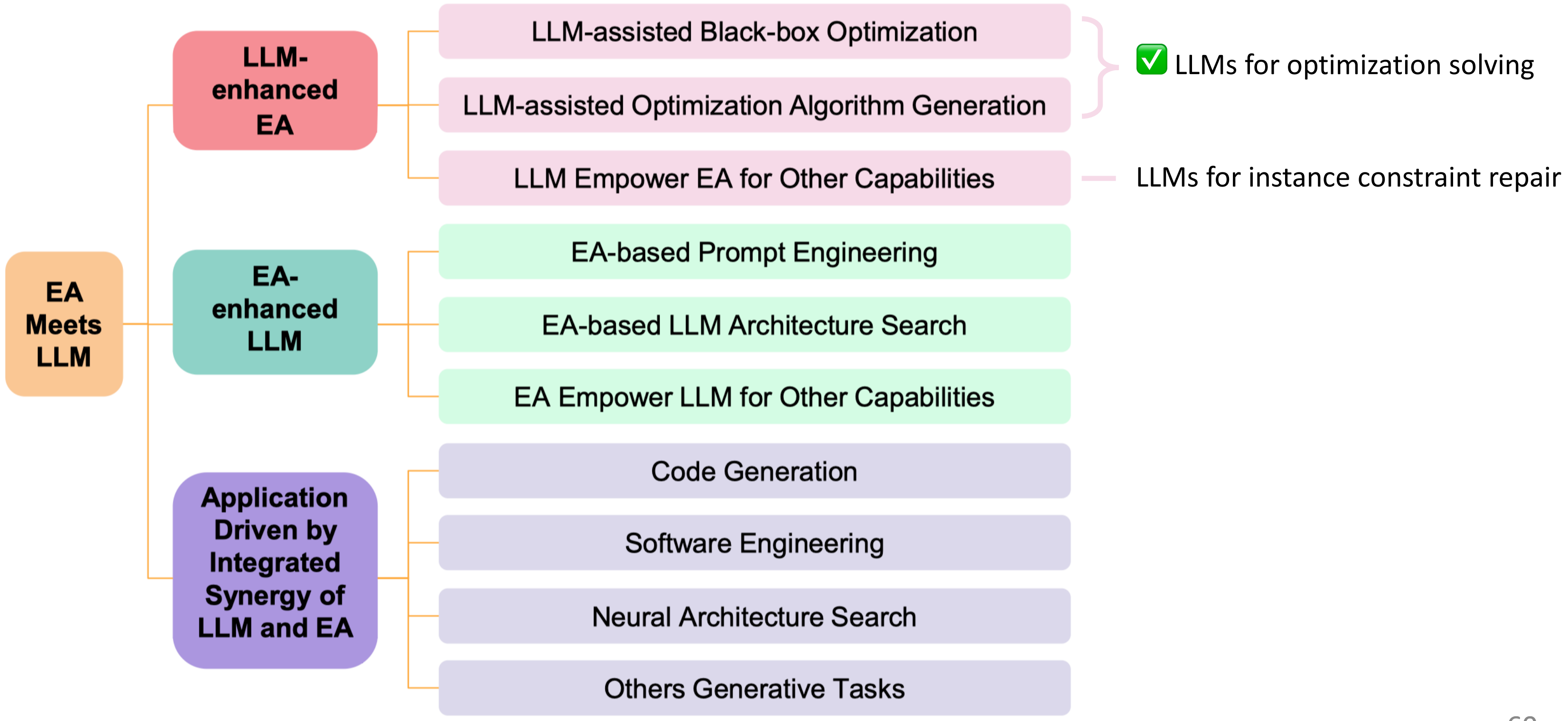
- **Rapid Progress of Large Models**
  - LLM capabilities have accelerated rapidly
  - Complex tasks are now within reach
  - “Impossible” problems are becoming solvable



(a) Breakthroughs on AI benchmarks from 2020 to 2025 (Yao et al., 2025)

Background

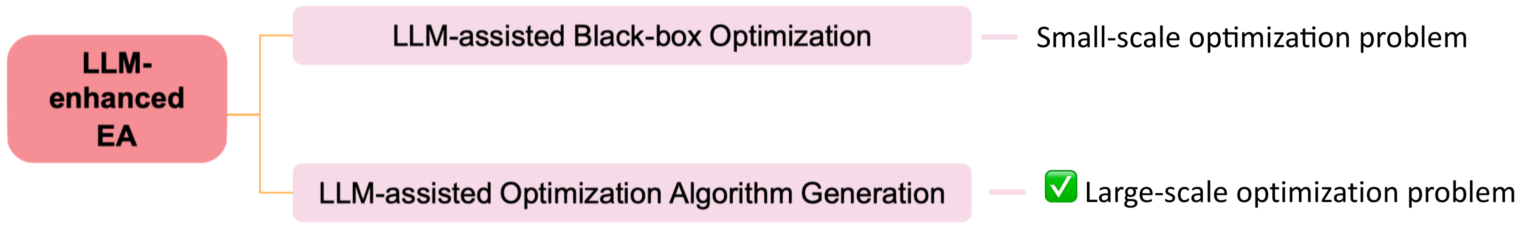
- **How LLMs Enhance Evolutionary Algorithms**
  - LLMs for optimization solving
    - Black-box optimization
    - Algorithm generation
  - LLMs for repairing infeasible optimization models


EA Meets LLM
LLM-enhanced EA
LLM-assisted Black-box Optimization
LLM-assisted Optimization Algorithm Generation
LLM Empower EA for Other Capabilities
EA-enhanced LLM
EA-based Prompt Engineering
EA-based LLM Architecture Search
EA Empower LLM for Other Capabilities
Application Driven by Integrated Synergy of LLM and EA
Code Generation
Software Engineering
Neural Architecture Search
Others Generative Tasks
✅ LLMs for optimization solving
LLMs for instance constraint repair

(a) Categorization of research works on the integration of LLMs and Evolutionary Algorithms (EAs)(Wu et al., 2024)

Background

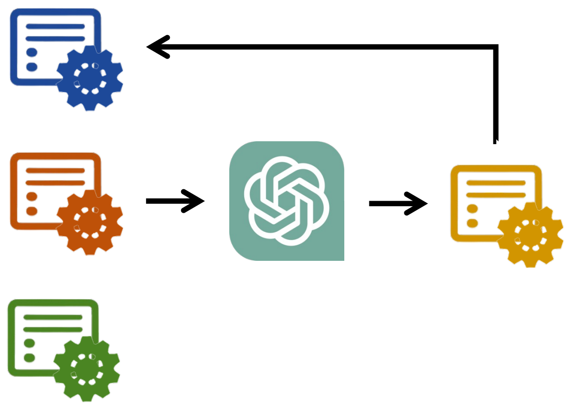
- **How LLMs Enhance Evolutionary Algorithms**
  - **Black-box Optimization**
    - LLM acts as a solver via dialogue
    - Or serves as a search operator
    - Easy to use, but limited by context & reasoning
    - ⚠️ Only suitable for small-scale problems
  - **Algorithm Generation**
    - LLM generates optimization algorithms
    - Algorithms are shorter than full problem inputs
    - Leverages LLM's code generation strength
    - ✅ Scalable to large-scale problems

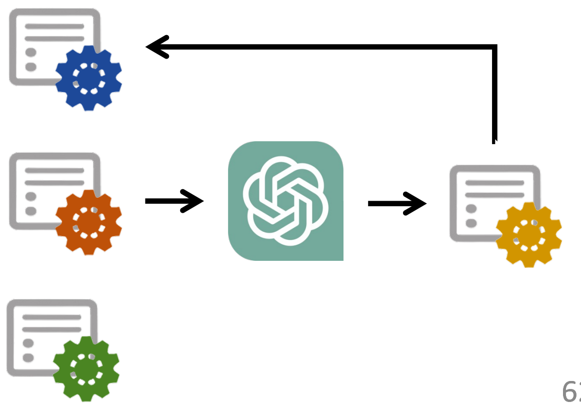

LLM-enhanced EA
LLM-assisted Black-box Optimization
Small-scale optimization problem
LLM-assisted Optimization Algorithm Generation
✅ Large-scale optimization problem

(a) Two main LLM-enhanced EA approaches for optimization solving(Wu et al., 2024)

Basic Method

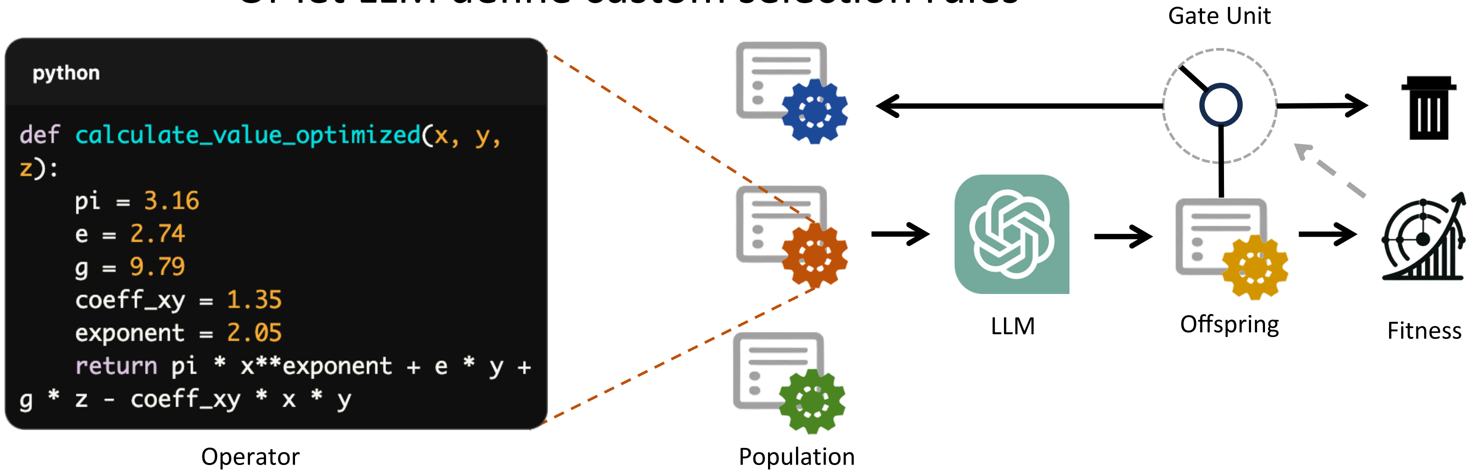
- **How EA Combines with LLM to Generate Optimization Code**
  - **Whole-code Evolution**
    - Directly evolve entire heuristic codes
    - Different paradigms may mix (e.g. Genetic Algorithm + Feasibility Pump)
    - ⚠️ Easy to use, but often unstable due to incompatible structures
  - **Operator-level Evolution**
    - Fix a high-quality heuristic framework (e.g. Large Neighborhood Search)
    - Use EA + LLM to evolve key operators (e.g. neighborhood selection)
    - ✅ More stable and effective


(a) Evolve the entire heuristic algorithm


(b) Evolve a component (e.g. operator)

Basic Method

- **Practical Workflow of EA + LLM for Code Evolution**
  - **Select Target**
    - Decide which operator to evolve (e.g., neighborhood selection)
    - Define its input and output interfaces
  - **Initialize Population**
    - Collect existing implementation strategies
    - Or generate initial candidates via LLM using interface descriptions
  - **Parent Selection**
    - Use standard EA techniques (e.g., tournament, roulette wheel)
    - Or let LLM define custom selection rules

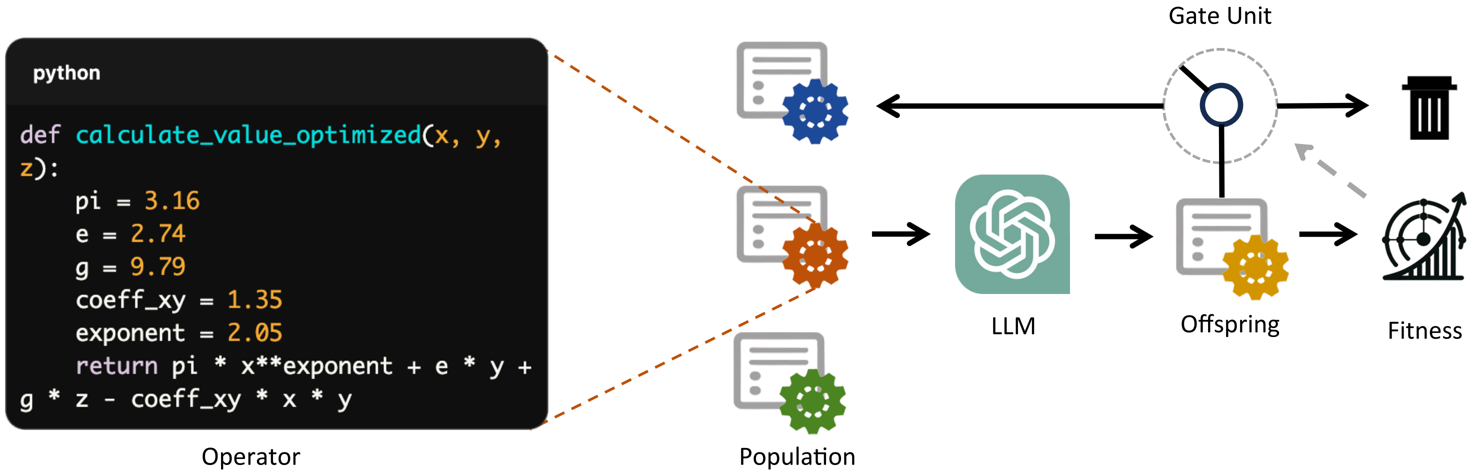


```python
def calculate_value_optimized(x, y,
z):
    pi = 3.16
    e = 2.74
    g = 9.79
    coeff_xy = 1.35
    exponent = 2.05
    return pi * x**exponent + e * y +
g * z - coeff_xy * x * y
```

(a) EA + LLM workflow for evolving heuristic operators

Basic Method

- **Practical Workflow of EA + LLM for Code Evolution**
  - **Generate Offspring**
    - Use LLM as a crossover/mutation operator
    - Input parent code, output new candidate strategy
  - **Evaluate and Filter**
    - Run offspring on tasks to compute fitness
    - Gate unit decides whether to keep, replace, or discard based on performance
  - **Eventually, you obtain an efficient, evolved operator** 🥳 🥳



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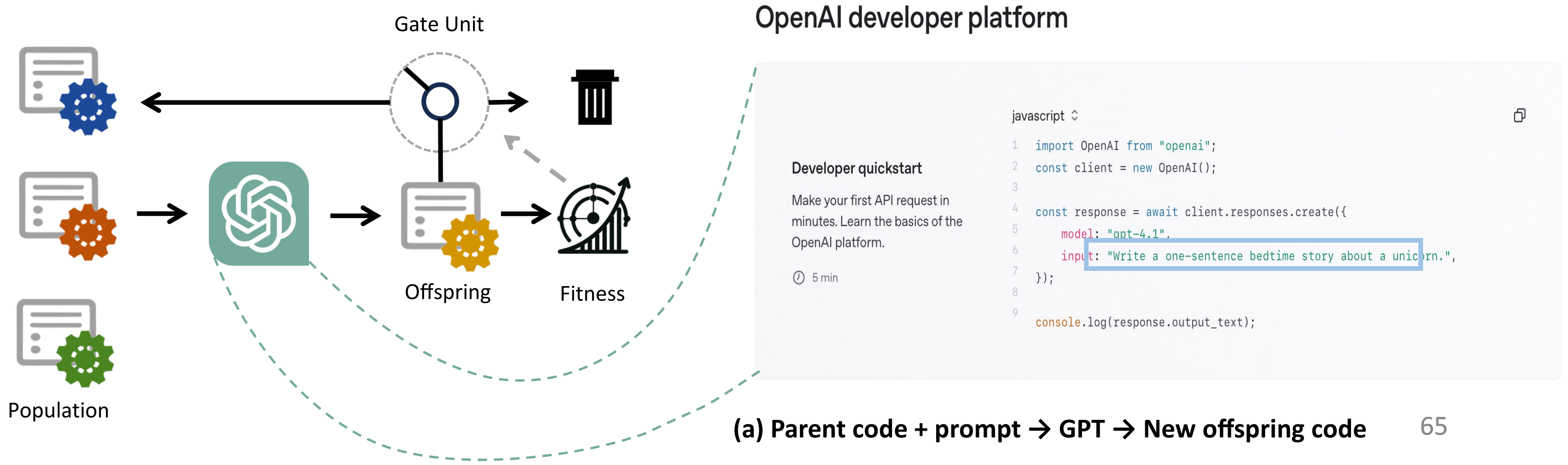
    exponent = 2.05
    return pi * x**exponent + e * y +
g * z - coeff_xy * x * y
```

(a) EA + LLM workflow for evolving heuristic operators

Basic Method

- **How to Use LLMs as Black-box Optimizers via API**
  - **Key Idea**
    - We use the LLM as a black-box optimizer
    - Just send the parent strategy + prompt to the API
    - and extract new offspring code from the response
  - **Example**
    - Using the OpenAI API (https://platform.openai.com/docs/overview)
    -

```js
input: "Improve this strategy: [parent_code_here]"
→ output: [new evolved code]
```



Developer quickstart

Make your first API request in minutes. Learn the basics of the OpenAI platform.

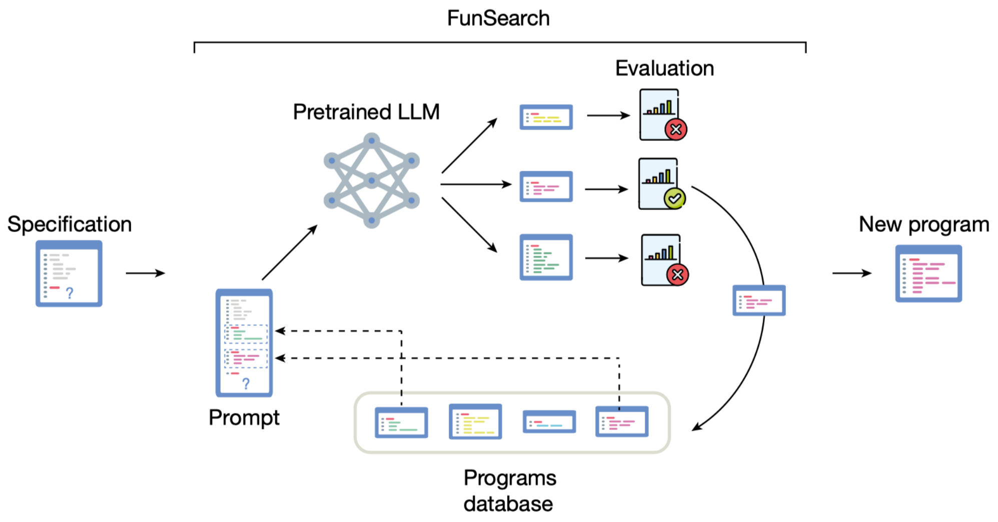
5 min

```javascript
import OpenAI from "openai";
const client = new OpenAI();

const response = await client.responses.create({
    model: "gpt-4.1",
    input: "Write a one-sentence bedtime story about a unicorn.",
});

console.log(response.output_text);
```

(a) Parent code + prompt → GPT → New offspring code

Advanced Method

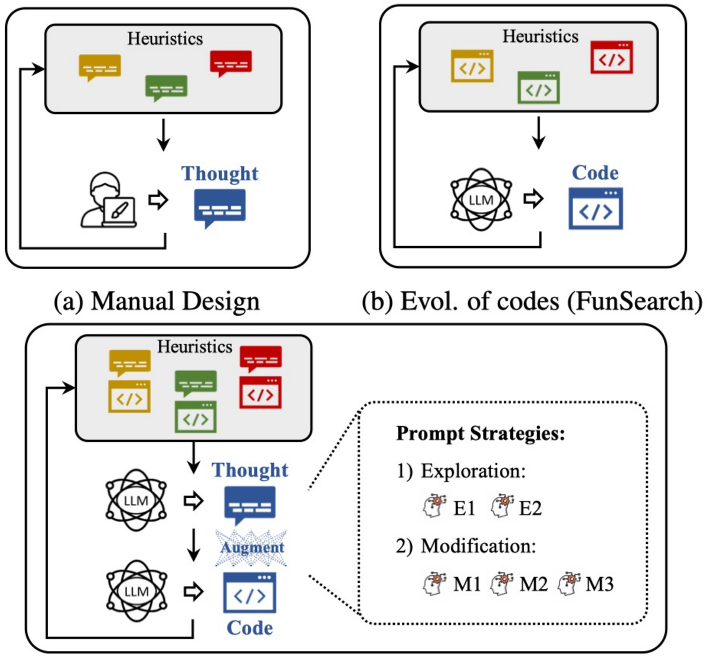
- **[Nature] Mathematical discoveries from program search with large language models**(Romera-Paredes et al., 2024)
  - **Ranked Prompting for Evolution**
    - Show 3 programs: A > B > C
    - → LLM learns what "better" looks like
  - **Island Model**
    - Independent evolution of subgroups
    - → Encourages diversity, avoids local optima
  - **Program Database**
    - Stores all correct programs
    - → Sampled to build prompts



(a) FunSearch framework (Romera-Paredes et al., 2024)

Advanced Method

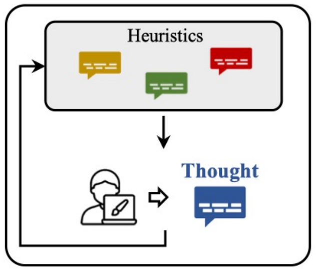
- **[ICML2024 Oral] Evolution of Heuristics: Towards Efficient Automatic Algorithm Design Using Large Language Model**(Liu et al., 2024)
  - 🚀 **Key Innovation: Co-evolution of Code and Thought**
  - **Before-**Most prior work (e.g., FunSearch) evolves code only
  - **EOH's Idea**-Evolve both:
    - 🧠 Natural language description ("thought"): summarizes the high-level idea
    - 💻 Code: implements the details
  - **Benefit**
    - → Thought helps understanding and generalization
    - → Code offers executable precision



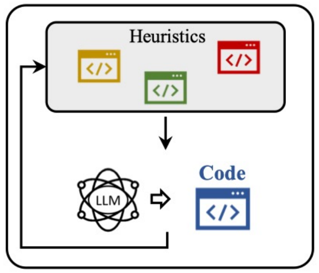
(a) EOH framework(Liu et al., 2024)

Advanced Method

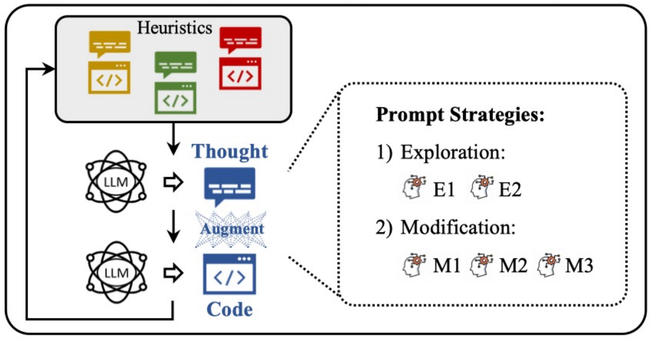
- **[ICML2024 Oral] Evolution of Heuristics: Towards Efficient Automatic Algorithm Design Using Large Language Model**(Liu et al., 2024)
    - **Prompts' Key Idea:**
        - Use LLM as an evolutionary operator with carefully designed prompt strategies→ Mimic how humans generate new ideas
    - ✨ **Two Categories of Prompts:**
        - Exploration (E-series)
            - E1 – "Create something new"
            - E2 – "Same idea, new form"
        - Modification (M-series)
            - M1 – Improve it
            - M2 – Tune it
            - M3 – Simplify it



(a) Manual Design



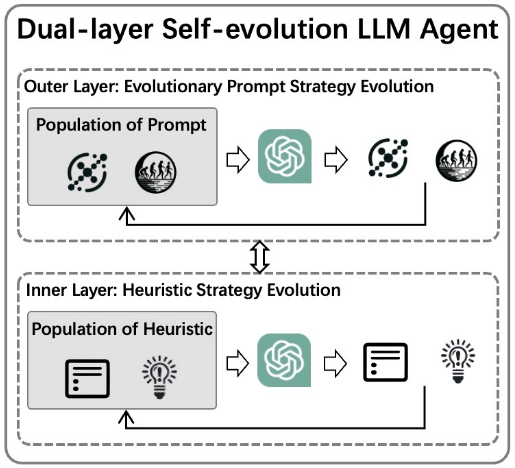
(b) Evol. of codes (FunSearch)



(a) EOH framework(Liu et al., 2024)

Advanced Method

- **[ICML2025 Spotlight] Large Language Model-driven Large Neighborhood Search for Large-Scale MILP Problems** (Ye et al., 2025)
    - ❌ **Problem in Prior Work**
        - Prompt strategies are fixed or handcrafted
        - Evolution only happens at the strategy level
        - Leads to:
            - 🧬 Limited diversity in generated heuristics
            - 🌀 Easy stagnation in local optima
    - ✅ **Key Innovation**
        - Dual-layer Self-evolutionary Structure


Dual-layer Self-evolution LLM Agent
Outer Layer: Evolutionary Prompt Strategy Evolution
Population of Prompt
Inner Layer: Heuristic Strategy Evolution
Population of Heuristic

(a) Dual-layer evolution

Layer	What It Evolves	Goal
🟠 **Outer Layer**	Prompt strategies *(how LLM evolves)*	**Exploration / Diversity**
🔵 **Inner Layer**	Heuristic strategies *(code + thought)*	**Exploitation / Convergence**

(b) Functional roles of the two layers in the self-evolutionary LLM agent

Advanced Method

- **[ICML2025 Spotlight] Large Language Model-driven Large Neighborhood Search for Large-Scale MILP Problems** (Ye et al., 2025)

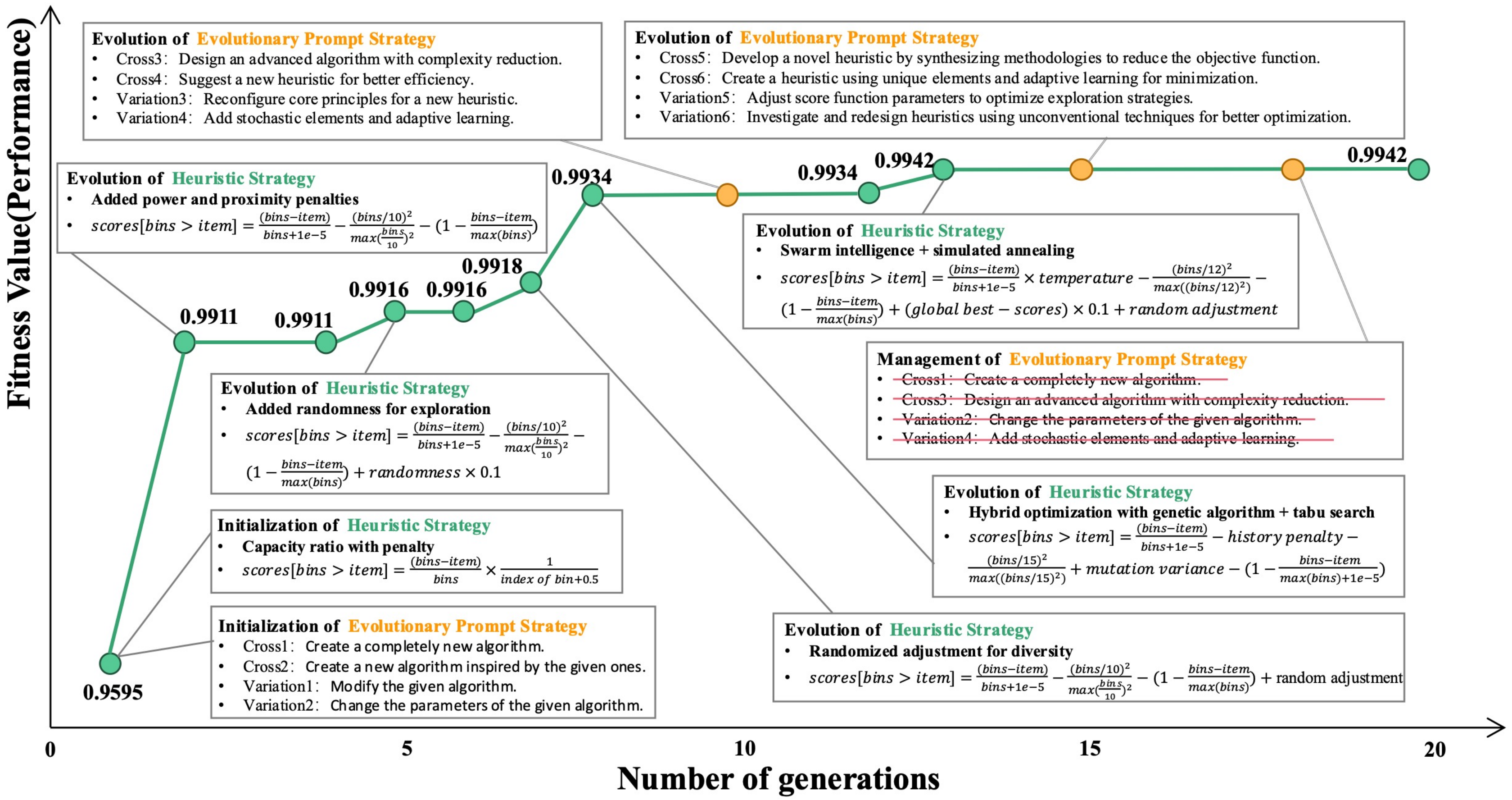
  ❌ Problem in Prior Work

    - LLMs don't know which direction to evolve
    - Because previous generations only give them:
        - ✖ Few examples
        - ✖ No clear contrast between good and bad → Evolution is blind and directionless

  - ✅ **Key Innovation**
    - 🧠 Inspired by-Large Language Models as Optimizers(Yang et al., 2024)
        - LLMs can optimize without gradients just by seeing solution + score pairs and reasoning over natural language
    - **Differential Memory for Directional Evolution**
        - Provide the LLM with:
            - Multiple strategies + both their scores and ranks
            - Natural language thoughts
        - → So it can learn from differences and evolve better offspring strategies

Advanced Method

- **[ICML2025 Spotlight] Large Language Model-driven Large Neighborhood Search for Large-Scale MILP Problems** (Ye et al., 2025)



Evolution of Evolutionary Prompt Strategy
- Cross3: Design an advanced algorithm with complexity reduction.
- Cross4: Suggest a new heuristic for better efficiency.
- Variation3: Reconfigure core principles for a new heuristic.
- Variation4: Add stochastic elements and adaptive learning.

Evolution of Evolutionary Prompt Strategy
- Cross5: Develop a novel heuristic by synthesizing methodologies to reduce the objective function.
- Cross6: Create a heuristic using unique elements and adaptive learning for minimization.
- Variation5: Adjust score function parameters to optimize exploration strategies.
- Variation6: Investigate and redesign heuristics using unconventional techniques for better optimization.

Evolution of Heuristic Strategy
- **Added power and proximity penalties**
- $scores[bins > item] = \frac{(bins-item)}{bins+1e-5} - \frac{(bins/10)^2}{max(\frac{bins}{10})^2} - (1 - \frac{bins-item}{max(bins)})$

Evolution of Heuristic Strategy
- **Swarm intelligence + simulated annealing**
- $scores[bins > item] = \frac{(bins-item)}{bins+1e-5} \times temperature - \frac{(bins/12)^2}{max((bins/12)^2)} - (1 - \frac{bins-item}{max(bins)}) + (global\ best - scores) \times 0.1 + random\ adjustment$

Evolution of Heuristic Strategy
- **Added randomness for exploration**
- $scores[bins > item] = \frac{(bins-item)}{bins+1e-5} - \frac{(bins/10)^2}{max(\frac{bins}{10})^2} - (1 - \frac{bins-item}{max(bins)}) + randomness \times 0.1$

Management of Evolutionary Prompt Strategy
- ~~Cross1: Create a completely new algorithm.~~
- ~~Cross3: Design an advanced algorithm with complexity reduction.~~
- ~~Variation2: Change the parameters of the given algorithm.~~
- ~~Variation4: Add stochastic elements and adaptive learning.~~

Initialization of Heuristic Strategy
- **Capacity ratio with penalty**
- $scores[bins > item] = \frac{(bins-item)}{bins} \times \frac{1}{index\ of\ bin+0.5}$

Evolution of Heuristic Strategy
- **Hybrid optimization with genetic algorithm + tabu search**
- $scores[bins > item] = \frac{(bins-item)}{bins+1e-5} - history\ penalty - \frac{(bins/15)^2}{max((bins/15)^2)} + mutation\ variance - (1 - \frac{bins-item}{max(bins)+1e-5})$

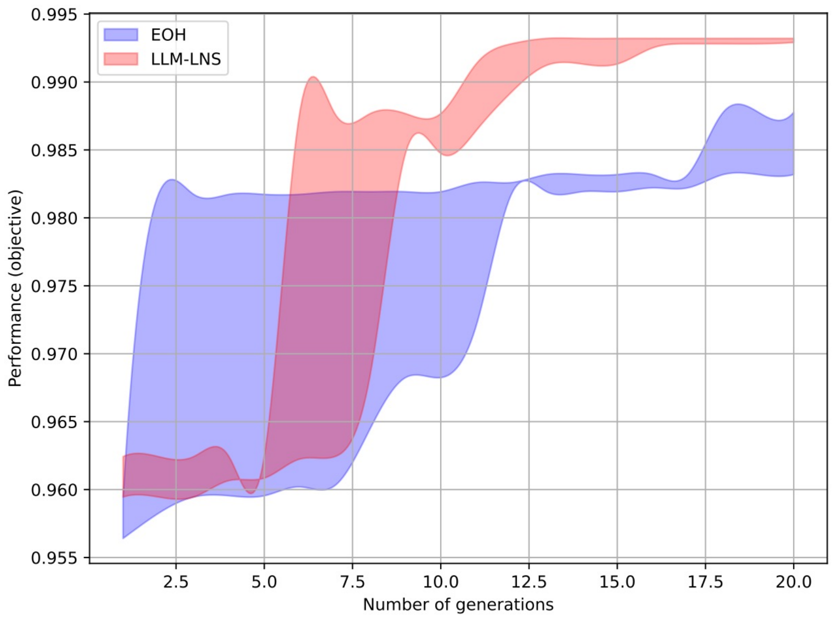
Initialization of Evolutionary Prompt Strategy
- Cross1: Create a completely new algorithm.
- Cross2: Create a new algorithm inspired by the given ones.
- Variation1: Modify the given algorithm.
- Variation2: Change the parameters of the given algorithm.

Evolution of Heuristic Strategy
- **Randomized adjustment for diversity**
- $scores[bins > item] = \frac{(bins-item)}{bins+1e-5} - \frac{(bins/10)^2}{max(\frac{bins}{10})^2} - (1 - \frac{bins-item}{max(bins)}) + \text{random adjustment}$

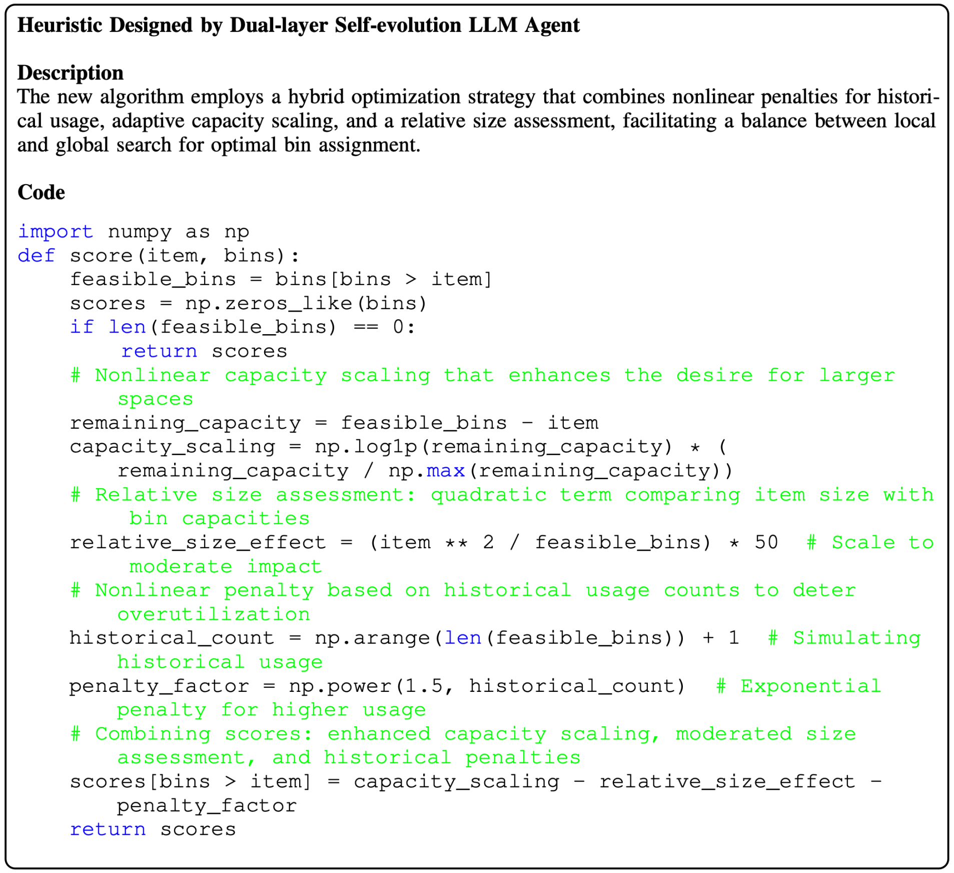
(a) Evolution of Dual-layer Self-evolutionary LLM Agent for online bin packing (Ye et al., 2025)

Advanced Method

- **[ICML2025 Spotlight] Large Language Model-driven Large Neighborhood Search for Large-Scale MILP Problems** (Ye et al., 2025)



(a) Evolutionary Progress of Heuristic Strategies in Online Bin Packing



Heuristic Designed by Dual-layer Self-evolution LLM Agent

Description

The new algorithm employs a hybrid optimization strategy that combines nonlinear penalties for historical usage, adaptive capacity scaling, and a relative size assessment, facilitating a balance between local and global search for optimal bin assignment.

Code

```
import numpy as np
def score(item, bins):
    feasible_bins = bins[bins > item]
    scores = np.zeros_like(bins)
    if len(feasible_bins) == 0:
        return scores
    # Nonlinear capacity scaling that enhances the desire for larger
       spaces
    remaining_capacity = feasible_bins - item
    capacity_scaling = np.log1p(remaining_capacity) * (
       remaining_capacity / np.max(remaining_capacity))
    # Relative size assessment: quadratic term comparing item size with
        bin capacities
    relative_size_effect = (item ** 2 / feasible_bins) * 50  # Scale to
        moderate impact
    # Nonlinear penalty based on historical usage counts to deter
       overutilization
    historical_count = np.arange(len(feasible_bins)) + 1  # Simulating
       historical usage
    penalty_factor = np.power(1.5, historical_count)  # Exponential
       penalty for higher usage
    # Combining scores: enhanced capacity scaling, moderated size
       assessment, and historical penalties
    scores[bins > item] = capacity_scaling - relative_size_effect -
       penalty_factor
    return scores
```

(b) Heuristic Designed by Dual-layer Self-evolution LLM Agent

Future Directions

- **Multimodal Optimization with LLMs**
    - Combine problem modeling and solution generation using multimodal inputs (e.g., text + code)
    - Enable end-to-end optimization pipelines via LLMs' multimodal understanding
- **Improving Diversity and Generalization**
    - Introduce continual learning mutation operators to adapt to changing problem spaces
    - Use performance-based feedback to evolve more effective mutation strategies
- **Modular Code Generation for Complex Logic**
    - Decompose complex logic into modular sub-tasks for better generation
    - Use interactive interfaces to guide LLMs and EAs in coordinated code generation

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Thanks

Any question?